

THE 1567/1260
ELEMENTS
OF
EUCLID.

*Explained and Demonstrated in a New and
most Easy Method.*

With the Uses of each

PROPOSITION
In all the Parts of the
MATHEMATICS.

By **Claude Francois Milliet** D.D.

Done out of **FRENCH**
Augmented, and Illustrated
per Plates, and the Effigies of

By **Reeve Williams**, Tailor-math.

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To the Honourable

Samuel Pepys, Esq;

Principal Officer

OF THE

N A V Y,

SECRETARY

TO THE

A D M I R A L T Y,

AND

PRESIDENT

OF THE

ROYAL SOCIETY.

Honoured Sir,

THE Countenance and
Encouragement, you
have always given to Ma-
thematical Learning, espe-

A 2

cially

Dedication.

cially as it hath a tendency
to Promote the Publick
Good ; has emboldened
me, humbly to Present your
Honour, with this little
Peace ; which hath the
Admirable Euclid for its
Author, and the Learned
D' Chales for the Commen-
tator, the excellency of the
Subject, with the Apt and
Profitable Application
thereof, in its Uses ; did
first induce me to Translate
it for my own use, the be-
nefit and quickening in
those

Dedication.

those Mathematical Studies, that some professed to have receiv'd, did prevail with me to make it Publick, and the great Obligations I lie under, for the many undeserv'd Favours of your Honour toward me; (I thought) did engage me on this occasion, to make some Publick Testimony and Acknowledgment thereof. I therefore humbly beg your Honours Patronage of this little Book, and your Par-

Dedication.

don for this Address, in-
treating you will be pleased
to look upon it with that
Benign Aspect, as you
have been pleased always
to vouchsafe to him who
is

Your Humble

and most

Obliged

Servant

Reeve Williams.



THE
Authors Preface
TO THE
REEDER.

HAVING long since observed, that the greatest part of those that learn *Euclid's Elements*, are very often dissatisfied therewith, because they know not the use of *Propositions* so inconsiderable in appearance; and yet so difficult: I thought it might be to good purpose, not only to make them as easie as possible, but also to add some *Uses* after each *Proposition*, to shew how they are applicable to Practice. And this hath obliged me to change some of the *Demonstrations*, which I looked upon to be too troublesome, and above the usual reach.

E. I. A 4.

The Author to the Reader.

reach of beginners, and to substitute others more intelligible. For this cause, I have Demonstrated the *Fifth Book* after a method much more clear than that, that makes use of *equimultiples*. I have not given all the *Uses* of the *Propositions*, for that would have made it necessary to recite all the *Mathematicks*, and would have made the Book too big, and too hard. Wherefore, I have only made choice of some of the plainest and easiest to conceive, I would not have you to stand too much upon them, nor make it your study to understand them perfectly, because they depend upon other principles besides, for which Reason I have distinguished them with the *Italick Letter*. This is the design of this small Treatise, which I willingly publish in a time when the *Mathematicks* are more than ever studied.

Millet D' Chales



THE

T H E

TRANSLATOR'S PREFACE.

TH E Reader having perused the Authors Preface, with this farther intimation, that he will find the Subject and Scope of this work succinctly and pertinently presented to him, in the Argument before each particular Book, may I presume, expect the less from me, and I shall not at all endeavour to bespeak the Readers acceptance of Euclid's Elements, or persuade him to believe, the Necessary and Excellent Usefulness thereof, because every mans Experience, so far as he understands them, is an abundant Testimony thereunto. Neither shall I need to commend the Method with the uses of our Author D' Chales (who is well known to the Learned of this Age by his

A. 5.

The Translator's Preface.

* *Cirius Mathe:*
his Navigation,
his Local Mo-
tion, &c.

his several excellent Mathe-
matical Tracts (for whoso-
ever shall be a while conversant
with this Book, may I presume
find that instruction and in-
couragement in the learning of Euclid's Ele-
ments as he hath not before met with in our
English Tongue, and this as it hath been my
own experience since I Translated it from the
French for the use of my English Schollars,
so it is one great cause of its coming abroad
into the World, for such as had learned by
it found it difficult to attain, unless Trans-
cribed, which they thought tedious, being a sub-
ject so Voluminous in Manuscript, and full of
Schemes, I did therefore at their Request, and
the importunity of some Friends condescend
to the Printing thereof, tho' not without much
overseesness to my own Mind, being unwilling
to expose wy self in any publick thing, in this
nice Critical Age; but that Difficulty being
now overcome, I shall only give the Reader
to understand, that I have faithfully ren-
dered this Peice into English, according to the
sense of the Author, but here and there o-
mitting some small Matters, which I judged
not so properly related to the Subject of this
Work, and therein will make no want to the
Reader, nor I hope be any offence to the In-
genious Author himself, I have only one
thing

The Translator's Preface.

thing more to add, and that upon the account of an Objection I have met with, that here is not all the Books of Euclid, and it is true, here is not all, here are only the first six Books, and the Eleventh and Twelfth, the other being purposely omitted by our learned Author, who judged the understanding of these to be sufficient for all the parts of the Mathematicks, * and I could also give Instances of other excellent Authors that are of his Opinion, and have taken the like Course, nay the truth is some very learned in the Mathematicks, have reduced the Propositions of these Books to a much less Number, and yet have thought them a compleat foundation to all the Sciences Mathematical, but I shall not trouble my Reader farther on this account, not doubting but when he hath perused and well considered our Euclid, he will have a better Opinion thereof, than any thing I can now say, may justly hope to beget in him, and so I shall submit my whole Concern herein to the Impartial Reader, and remain ready to serve him,

* See Argument before the Eleventh Book, Page 304.

Reeve Williams.

From my School at the
Virginia Coffee-house
in St. Michael's Alley
in Cornhill.

A. D.

ADVERTISEMENT.

WHereas in the *French*, all the Definitions which needed all the Propositions, as also all those Uses, our Author thought fit to illustrate by Schemes, were done in the Book in Wood, here they are in Copper Plates, to be placed at the end of the Book. The Definitions and Uses are in the Plates marked with Arithmetical Characters or Ziphers; but the Propositions in Alphabetical Ziphers at the beginning of the Book, or at least when you come to the first Definition, you are referred to the number of the Plate, in which you shall find the Scheme proper thereto, as also the Def. Prop. and Uses belonging unto the Book, unless the Plate could not contain them, and then you are referred to the next Plate; where on the head thereof you shall find the Book it belongeth to, and the Propositions or Uses, continued in their Order.



THE

EIGHT BOOKS

OF

Euclid's *Elements*:

With the Uses of each

PROPOSITION.

The FIRST BOOK.

EUCLID's Design in this Book is to give the first Principles of Geometry; and to do the same Methodically, he begins with the Definitions, and Explication of the most ordinary Terms, then he exhibits certain Suppositions, and having proposed those Maxims which natural Reason teacheth, he pretends to put forward nothing without Demonstration, and to convince any one which will consent to nothing but what he shall be obliged to acknowledge; in his first Propositions he treateth of
Lines,

The First Book of

Lines, and of the several Angles made by their intersecting each other, and having occasion to Demonstrate their Properties, and compare certain Triangles, he doth the same in the First Eight Propositions; then teacheth the Practical way of dividing an Angle and a Line into two equal parts, and to draw a Perpendicular he pursues to the propriety of a Triangle; and having shewn those two Parallel Lines, he makes an end of the Explication of this First Figure, and passeth forwards to Parallelograms; giving a way to reduce all sorts of Polygons into a more Regular Figure: He endeth this Book with that Celebrated Proposition of Pythagoras, and Demonstrates, that in a Rectangular Triangle, the Square of the Base is equal to the sum of the Squares of the Sides, including the Right Angle.

DEFINITIONS.

1. **A** Point is that which hath no part.

This Definition is to be understood in this sence. The quantity which we conceive without distinguishing its parts, or without thinking that it hath any, is a Mathematical Point far differing from those of Zeno, which were altogether indivi-



invisible, since one may doubt with a great deal of Reason, if those last be possible, which yet we cannot of the first, if we conceive them as we ought.

2. A Line is a length without breadth.

The sense of this Definition is the same with that of the foregoing, the quantity which we consider, having length without making any reflection on its breadth, is that we understand by the word Line, although one cannot draw a real Line which hath not a determinate breadth, it is generally said that a Line is produced by the motion of a Point, which we ought well to take notice of, seeing that by a motion after that manner may be produced all sorts of quantities; imagine then that a Point moveth, and that it leaveth a trace in the middle of the way which it passeth, the Trace is a Line.

3. The Two ends of a Line are Points.

4. A straight Line is that whose Points are placed exactly in the midst, or if you would rather have it, a straight Line is the shortest of all the Lines which may be drawn from one Point to another.

5. A Superficies is a quantity to which is given length and breadth without considering the thickness.

6. The extremities of a Superficies are Lines.

7. A plain or straight Superficies is that whose Lines are placed equally between the extremities, or that to which a straight Line may be applied any manner of way.

Plate I.
Fig. 1.

I have already taken notice that motion is capable of producing all sorts of quantity, whence we say that when a Line passeth over another, it produces a Superficies or a Plain, and that that motion hath a likeness to Arithmetical Multiplication; imagine that the Line AB moveth along the Line BC , keeping the same situation, without inclining one way or the other, the Point A shall describe the Line AD , the Point B the Line BC ; and the other Points between other Parallel Lines, which shall compose the Superficies $ABCD$. I add, that this motion corresponds with Arithmetical Multiplication, for if I know the number of Points which are in the Lines AB , BC , Multiplying of them one by the other. I shall have the number of Points which Composeth the Superficies $ABCD$; as if AB contains four points, and BC six; saying, Four times Six are Twenty Four, the Superficies AB , CD , should be Composed of Twenty Four Points.

Points: Now I may take for a Mathematical Point any quantity whatsoever, for Example a Foot, provided I do not subdivide the same into Parts.

8. A plain Angle is the opening of Two Lines which intersect each other, and which Compose not one single Line.

As the opening D, of the Lines AB, Fig. 2. CB; which are not parts of the same Line.

A Right Lined Angle is the opening of two streight Lines.

It is principally this sort of Angles which I intend to treat of at present, because experience doth make me perceive, that the most part of those who begin, do mistake the measuring the quantity of an Angle, by the length of the Lines which Compose it the same.

The most open Angle is the greatest, Fig. 3, 4. that is to say, when the Lines including an Angle are farther asunder than those of another Angle, taking them at the same distance from the Points of intersection of their Lines, the first is greater than the Second; so the Angle A is greater than E; because if we take the Points B and D, as far distant from the Point A, as the Points G and L are from the Point E; the Points B and D are farther asunder than the Points G and L;
from

from whence I conclude, that if EG, EL, were continued, the Angle E would be of the same Measure, and less than the Angle A.

We make use of Three Letters to express an Angle, and the Second Letter donotes the Angular Point, as the Angle BAD, is the Angle which the Lines BA, AD, doth form, at the Point A; the Angle BAC, is that which is formed by the Lines BA, AC; the Angle CAD is comprehended under the Line CA, AD.

Fig. 3.

The Arch of a Circle is the measure of an Angle, thus designing to measure the quantity of the Angle BAD, I put one Foot of the Compasses on the Point A, and with the other I describe an Arch of a Circle BCD, the Angle shall be the greater by how much the Arch BCD, which is the measure thereof, shall contain a greater portion of a Circle, and because that commonly an Arch of a Circle is divided into Three Hundred and Sixty equal Parts, called Degrees. It is said that an Angle containeth Twenty, Thirty Forty Degrees, when the Arch included betwixt its Lines, contains Twenty, Thirty, Forty Degrees; so the Angle is greatest, which containeth the greatest number of Degrees: As the Angle BAD,

is

is greater than GEL , the Line CA divideth the Angle BAD , in the middle, because the Arches BC , CD , are equal, and the Angle BAC is a part of BAD , because the Arch BC is part of the Arch BD .

10. When a Line falling on another Line, maketh the Angle on each side thereof equal: Those Angles are Right Angles, and the Line so falling is a Perpendicular.

As if the Line AB falling on CD , Fig. 5. makes the Angles ABC , ABD equal, that is if having on B as a Center, described a Semicircle CAD , the Arches AC , DA are found equal, the Angle ABC , ABD , are called Right Angles, and the Line AB a Perpendicular. Now because the Arch CAD is a Semicircle, the Arch CA , AD , are each of them a Quadrant, that is to say the Fourth part of the Three Hundred and Sixty Degrees, which is Ninety.

11. An Obtuse Angle is greater than a Right Angle.

As the Angle EBD is Obtuse or Blunt, Fig. 5. because its Arch EAD contains more than a Quadrant.

12. An Acute Angle is less than a Quadrant.

As the Angle EBC is Acute, because Fig. 5. the

the ArchEC; which is the measure thereof, is less than Ninety Degrees.

13. A Term is the extremity or end of a Quantity.

14. A Figure is a Quantity terminated by one or many Terms.

It ought to be bounded and inclosed on every side to be called a Figure.

15. A Circle is a Plain Figure, whose bounds are made by the winding or turning of a Line, which is called Circumference, and which is equally distant from the middle Point.

Fig. 6.

The figure RVSX is a Circle, because all the Lines TR, TS, TV, TX, drawn from the Point T, to the Line RVSX, are equal.

16. This middle Point is called the Center.

Fig. 7.

17. The Diameter of a Circle is any Line whatsoever which passeth through the Center, and which ends at the Circumference, cutting the same into two equal parts.

As the Line VTX, or RTS; some one may perhaps doubt whether the Line VTX doth effectually divide the Circle into Two equally, that is, whether the part VSX, be equal to the part VRX; that they are, I thus prove,

Fig. 7.

Let it be imagined that the part VRX, be

be placed on VSX , I say that the one shall not exceed the other; for if the one as VSX , should exceed the other VRX , the Line TR would then be shorter than TS , the same of TY , in respect of TZ , which is contrary to the Definitions of a Circle; that is, that all the Lines drawn from the Center of a Circle to the Circumference, be all equal.

18. A Semicircle is a Figure Termined by the Diameter and half the Circumference, as VSX .

19. Right Lined Figures are Termined by streight Lines, there are some of Three, of Four, of Five, or as many sides as you would have them.

Euclid distinguisheth Triangles by their Angles, or by their Sides.

20. An Equilateral Triangle is that whose Three sides are equal, as ABC .

21. An Isosceles Triangle is that which hath two equal Sides.

As if the Sides AB , AC , are equal, the Triangle ABC is an Isosceles Triangle.

22. Scalenum is a Triangle whose three Sides are unequal, as DEF .

23. A Right Angled Triangle is that which hath one Angle right, as DEF , supposing the Angle E to be right.

24. An Amblygonium Triangle hath one Angle Obtuse, as GHI .

25.

Fig. 10.

25. An Oxygoniutn Triangle hath all its Angles Acute, *as ABC.*

Fig. 8.

26. A Rect-Angle is a figure having four Sides, which hath all its Angles right.

Fig. 11.

27. A Square hath all its Sides equal and all its Angles right, *as AB.*

Fig. 12.

28. A Rect-Angular figure whose Lines are unequal and Angles right, is called an Oblong or long Square, *as CD.*

Fig. 13.

29. A Rhombus or Diamond figure, is that which hath four equal Sides, but is not right Angled, *as EF.*

Fig. 14.

30. A Rhomboides or Diamond like figure, is that whose opposite Sides and opposite Angles are equal, but hath neither equal Sides nor Right Angles, *as GH.*

31. Other Irregular Figures having four sides, are called Trapeziums.

Fig. 15.

32. Parallel or Equidistant Right Lines, are such which being in the same Superficies, if infinitely produced would never meet, *as the two Lines AB, CD.*

Fig. 15.

33 A Parallelogram is a figure whose opposite Sides are parallel, *as the figure AB, CD, of which the Sides AB, CD, AC, BD, are parallel.*

Fig. 16.

34. The Diameter of a Parallelogram is a streight Line drawn from Angle to Angle, *as BC.*

25. The Complements are the two lesser Parallelograms, through which the Diameter passeth not, *AFEH*, *GDIE*. Fig. 16.

Postulata or SUPPOSITIONS.

1. **I**T is supposed that a streight Line may be drawn from any Point whatsoever, to any other Point.
2. That a streight Line may be infinitely continued.
3. That on a Center given a Circle may be described, with any distance taken between the Compasses.

AXIOM S.

1. **Q**uantities or things equal to the same third, are also equal among themselves.
2. If to two equal quantities be added two other Quantities which are also equal, those Quantities which are produced shall be equal.
3. If from two equal Quantities be taken away two other equal Quantities, those Quantities remaining shall be equal.
4. If equal Quantities be added to unequal Quantities, their Sums shall be unequal.
5. If from equal Quantities be taken away unequal Quantities the Quantities remaining shall be unequal.
6. The Quantities, which are double, treble,

treble, quadruple, to the same, are equal among themselves.

7. Quantities are equal, when applied together, the Parts of the one are not extended beyond those of the other.

8. Lines and Angles are equal, when the one being applied to the other they agree.

9. Every whole is greater than its parts, and equal to all its parts taken together.

10. All Right Angles are equal amongst themselves.

Fig. 17.

Let there be proposed two Right Angles ABC, EFH . I say they are equal to each other; for if two equal Circles be described, CAD, HEG , on the Centres B and F , the quarters of Circles CA, HE , which are the measures of the Angles, ABC, EFH , shall be equal; therefore the Angles ABC, EFH whose measures being equal, are consequently equal.

Fig. 18.

The Eleventh maxim of *Euclid* beareth this signification, that if the Lines $A B, C D$, maketh with the Line $E F$, which cutteth both the said Lines, the Interminate Angle BEF, DFE , less than two Right Angles, the Lines $A B, C D$, being continued, shall meet each other on the same side with B and D .

Although this Maxim be true, yet it is not evident enough to be received for a maxim, wherefore I substitute another in the place thereof.

11. If

11. If two Lines are parallel, all the Perpendiculars inclosed between them shall be equal.

As if the Line AB , CD , are paral- Fig. 19.
lels, the perpendicular Lines EE , HG ,
are equal, for if EF was greater than
 GH , the Lines AB and CD should be
farther distant from each other towards
the Points E and F , than towards G
and H , which would be contrary to the
Definition of parallels, which sayeth that
they are equidistant from each other.

12. Two Right Lines do not com-
prehend a Superficies, that is to say, do
not encompass a space on every side.

13. The streight Lines have not the
same common Segment.

I would say, that of two streight Lines Fig. 20.
 AB , CD , which meeteth each other in
the Point B , is not made one single Line
 BD , but that they cut each other, and
separate after their so meeting.

For if a Circle be described on the
Center B , AFD shall be a Semicircle;
seeing the Line ABD passing through
the Center B , divideth the Circle into
two equally; the Segment CFD should
be also a Semicircle, if CBD were a
streight Line, because it passeth through
the Center B ; therefore the Segment
 CFD should be equal to the Segment
 B AFD ,

*AFD, a part as great as the whole ;
which would be contrary to the Ninth
Axiom.*

ADVERTISEMENT.

WE have two sorts of Propositions, some whereof considereth only a truth without descending to the practice thereof, and we call those Theorems.

The other proposeth something to be done or made, and are called Problems.

The first numbers of Citations is that of the Proposition, the second that of the Book: As by the 2 of the 3, is signified the second proposition of the third Book, but if one meet with one number, thereby is meant the Proposition of the Book, whose Explication is in hand.

PROPOSITION I.

PROBLEM.

UPon a finite Right Line *AB*, to describe an Equilateral Triangle *ACB*.

From the Centers *A* and *B*, at the distance of *AB*, describe the Circles cutting each other in the Point *C*; from whence draw two Right Lines *CA*, *CB*; thence are *AC*, *AB*, *BC*, *AC*, equal; wherefore the Triangle *ACB* is equilateral, which was to be done.

USE.

USE.

Euclid has not applyed this Proposition to any other use, but to demonstrate the two following Propositions, but we may apply it to the measuring of an inaccessible Line.

As for Example, let AB be an inaccessible Line, which is so by reason of a River or some other Impediment; make an Equaliteral Triangle, as BDE, on Wood, or Brass; or on some other convenient thing, which having placed Horizontally at a station at B, look to the Point, A, along the Side BD, and to some other Point C, along the side BE; then carry your Triangle along the Line BC, so far, that is untill such time as you can see the Point B, your first station by the side CG, and the Point A by the Side CF: I say that then the Lines CB and CA are equal; wherefore, if you measure the Line BC, you will likewise know the length of the Line AB.

PROPOSITION II.

PROBLEM.

AT a Point given A, to make a Right Line AG, equal to a Right Line given BC.

From the Center C, at the distance

B 2

CB,

CB, describe the Circle CBE, joyn AC; upon which raise the Equilateral Triangle ADC, produce DC to E, from the Center D, and the distance DE; describe the Circle DEH. Let DA be produced to the Point G in the Circumference thereof, then AG is equal to CB; for DG is equal to DE, and DA to DC; wherefore AG, CE, BC, AG, are equal; which was to be done.

S C H O L.

THe Line AG might be taken with a pair of Compasses; but the so doing answers no Postulate, as Proclus well intimates.

PROPOSITION III.

PROBLEM.

TWo Right Lines, A and BC, being given, from the greater BC, to take away the Right Line BE, equal to the lesser A.

At the Point B, draw the Right Line BD equal to A, the Circle described, from the Center B, at the distance BD, shall cut off BE, equal to BD, equal to A; equal to BE; which was to be done.

The use of the two preceding Propositions are very evident, since we are obliged

liged very often in our Geometrical Practices to draw a Line equal to a Line given; or to take away from a greater Line given, a part equal to a lesser.

PROPOSITION IV.

THEOREM.

IF Two Triangles ABC , EDF , have two Sides, of the one BA , AC , equal to two Sides of the other ED , DF , each to his correspondent side; that is BA to ED , and AC to DF ; and have the Angle A equal to the Angle D , contained under the equal Right Lines, they shall have the Base BC equal to the Base EF , and the Triangle BAC shall be equal to the Triangle EDF ; and the remaining Angles B , C , shall be equal to the remaining Angles E , F ; each to each, which are subtended by the equal Sides.

If the Point D be applied to the Point A , and the Right Line DE placed on the Right Line AB , the Point E shall fall upon B , because DE is equal to AB ; also the Right Line DF shall fall upon AC ; because the Angle A is equal to D : Moreover the Point F shall fall on the Point C , because AC is equal to DF : Therefore the Right Lines EF , BC , shall agree, because they have the same terms; and so consequently are e-

qual; wherefore the Triangle BAC is equal to DEF, and the Angles B, E; as also the Angles C, F do agree, and are equal, which was to be demonstrated.

USE.

USE 4.

Suppose I was to measure the inaccessible Line AB, I look from the Point C to the Point A and B, then I measure the Angle C thus; I place a Board or Table Horizontally, and looking successively with a Ruler towards the Point A and B; I draw two Lines making the Angle ACB, then I measure the Lines AC and BC, which I suppose to be accessible; I turn about my Board or Table towards some other place in the Field, placing it again Horizontally at the Point F; and looking along those Lines I have drawn on my Table, I make the Angle DEF equal to the Angle C; I make also FD, FE, equal to CA, CB: Now according to this Proposition the Lines AB, DE, are equal, wherefore whatsoever the Length of the Line DE is found to be, the same is the measure of the inaccessible Line AB.

Another USE may be this.

USE 4.

Suppose you were at a Billiard Table, and you would strike a Ball B by reflection,

fection, with another Ball A: Admit CD be one Side of the Table, now imagine a perpendicular Line BDE, I take the Line DE equal to DB, I say, that if you aim and strike your Ball A directly towards the Point E, the Ball A meeting the Side of the Table at F, shall reflect from thence to B, for in the Triangle BFD, EFD, the Side FD is common, and the sides BD, DE, equal, the Angles BED, EFD equal by the Proposition, the Angles AFC, DEF, being opposite are also equal, as I shall demonstrate hereafter, therefore the Angle of incidence AFC, is equal to the Angle of reflection BED and by consequence the reflection will be from AF to FE.

PROPOSITION V.

THEOREM.

IN every Isosceles Triangle, the Angles which are above the Base are equal, as also those which are underneath.

Let ABC be an Isosceles Triangle, viz. that the side AB, AC be equal, I say that the Angle ABC, ACB, are equal, as also the Angles GBC, HCB, which ly under the Base BC: Let there be imagined another Triangle DEF, having an Angle D equal to the Angle A, and the Sides DE, DF, equal to AB, AC: Now since the Sides AB,

AC, are equal, the Four Lines AB, AC, DE, DF, shall be equal.

Demonstration, because the Sides AB, DE, AC, DF, are equal, as also the Angles A and D, if we put the Triangle DEF on the Triangle ABC, the Line DE shall fall upon AB and DF, on AC and FE, on BC; (by the Fourth) therefore the Angle DEF shall be equal to ABC, and since one part of the Line DE falls on AB, the whole Line DE shall fall on AC, otherwise Two straight Lines would contain a space, therefore the Angle DEF shall be equal to the Angle GBC: Now if you should turn the Triangle DEF, and make the Line DE to fall on AB, and DF, on AC; because the Four Lines AB, DF, AC, DE, are equal, as also the Angles A and D; the Two Triangles ABC, DEF, shall ly exactly on each other, and the Angles ACB, DEF, HCB, IEF, shall be equal: Now according to our first Comparison, the Angle ABC was equal to DEF, and GBC to IEF, therefore the Angles ABC, ACB, which are equal to the same DEF, and GBC, HCB, which are also equal to the same IEF, are also equal among themselves.

I thought fit not to make use of Euclid's Demonstration, because it being too difficult

is difficult for young Learners to Apprehend, they are often discouraged to proceed.

PROPOSITION VI.

THEOREM.

IF Two Angles of a Triangle are equal, that Triangle shall be an Iſoſceles Triangle.

Let the Angles ABC , ACB , of the Triangle ABC be equal; I affirm, that their opposite Sides AB , AC , are equal to each other; let the Triangle DEF , have the Base EF equal to BC , and the Angle DEF , equal to ABC ; as also DFE , equal to ACB ; ſince then that we ſuppoſe that the Angles ABC , ACB , are equal; the Four Angles ABC , ACB , DEF , DFE , are equal: Now let us imagine the Baſe EF be ſo put on BC , that the Point E fall on B ; it is evident, that ſince they be equal, the one ſhall not exceed the other anywiſe; moreover the Angle E being equal to the Angle B , and the Angle FD to the Angle C , the Line EB ſhall fall on BA , and FD on CA ; therefore the Lines ED , FD , ſhall meet each other in the Point A ; from whence I conclude, that the Line ED is equal to BA .

Again let us turn over the Triangle DEF , placing the Point E on C , and

F on B; which must so happen, because BC is supposed equal to EF; and because the Angles F and B, E and C, are supposed equal, the Side FD shall fall on BA, and ED on CA, and the Point D on A; wherefore the Lines AC, DE, shall be equal; from whence I conclude, that the Sides AB, AC, are equal to each other, since they are equal to the same Side DE.

U S E.

USe 6.

THis Proposition is made use of, in measuring all inaccessible Lines, it is said that Thales was the first that measured the Height of the Egyptian Pyramids by their Shadows, it may easily be done by this Proposition; for if you would measure the Height of the Pyramid AB, you must wait till the Sun be elevated 45 degrees above the Horizon. (that is to say) until the Angle ACB be 45 degrees; now by the Proposition, the shadow BC is equal to the Pyramid AB, for seeing the Angle ABC is a Right Angle, and that the Angle ACB is half Right or 45 degrees, the Angle CAB shall also be half a Right Angle, as shall be prov'd hereafter. Therefore the Angles BCA, BAC, are equal, and by the Sixth, the Sides AB, BC, are also equal, I might measure the same without making use of the Shadow, by
going

going backwards from B, until the Angle, ACB be half a Right Angle, which I may know by a Quadrant: These Propositions are made use of in Trigonometry and several other Treatises.

I Omit the Seventh, whose use is only to Demonstrate the Eighth.

PROPOSITION VIII.

THEOREM.

IF Two Triangles be equally Sided, they shall also have their opposite Angles equal.

Let the Sides GI, LT; HI, VT; GH, LV, be equal: I say that the Angle GH shall be equal to the Angle LTV, IGH, to the Angle L, IHG, to the Angle V. From the Center H, at the distance HI, let the Circle IG be described, and from the Center G, with the Extent GI, let the Circle HI be described.

Demonstration. If the Base LV be laid on the Base HG, they will agree, because they are equal, I further add, that the Point T shall fall precisely at the Circumference of the Circle IG; since we suppose the Lines HI, VT are equal, it must likewise fall at the Circumference IH, seeing GI is equal to LT, therefore it shall fall on the Point I, which is the Point where those Circles

cut each other, but if you deny it, and say it should fall on some other Point, as at O, then I say the Line HO (that is to say VT) would be greater than HI, and the Line GO (that is to say LV) would be less than GI, which is contrary to the Hypo. from whence I conclude the Triangles do agree in all their parts; and that therefore the Angles GIH, is equal to the Angle LTV.

This Proposition is necessary for establishing the next following; moreover when we cannot measure an Angle, which is made in a Solid Body, by Reason we cannot place our Instruments; we take the Three Sides of a Triangle, and make another on Paper, whose Angles we easily measure, this is very useful in Dialling, and in cutting of Stone, and Bewelling of Timber.

PROPOSITION IX

PROBLEM

TO Bisect or Divide into two equal parts a Right-Lined Angle given.
SAY.

Take AS equal to AT, and draw the Line ST, upon which make an equilateral Triangle SVT; draw the Right Line VA, it shall Bisect the Angle.

Demonstration. AS is equal to AT, and

and the Side AV is common, and the Base SV equal to VT, therefore the Angle SAV is equal to TAV, which was to be done.

Q.E.D. signifieth Q.E.D.

This Proposition is very useful to Divide a Quadrant into Degrees, it being the same thing to Divide an Angle, or to Divide an Arch into two equal parts; for the Line AV divides as well the Arch ST as the Angle SAT, for if you put the Semi-Diameter on a Quadrant, you cut off an Arch of 60 Degrees, and Dividing that Arch into Two Equal parts, you have 30 Degrees, which being again Divided into two equal parts, you have 15 Degrees. It is true, to make an end of this Division you must Divide an Arch in Three, which is not yet known to Geometricians. Our Sea Compasses are Divided into 32 Points by this Proposition.

PROPOSITION XI

PROBLEM

To Divide a Line given into Two Equal parts.

Let the Line AB be proposed to be Divided into Two equal parts, erect an Equilateral Triangle ABC, (by the 1st) Di-

Divide the Angle ACB into two equal parts, (by the 9th.) I say the Line AB is Divided into two equal parts in the Point E : viz. That the Lines AE , EB , are equal.

Demonstration. The Triangle ACE , BCE , have the Side CE common, and the Sides CA , CB , equal, since the Triangle ACB is Equilateral: The Angles ACE , BCE , are equal, because the Angle ACB was Divided into two equal parts; therefore (by the 4th.) the Bases AE , EB , are equal.

U S E.

THis Proposition is often made use of, our ordinary Practice obligeth us thereto. Wherefore Geometricians would that a Line should be divided at once, and not by several Trials, and that by an infallible method, the Practice of this is most useful in Dividing of Measures into smaller parts.

PROPOSITION XL.

PROBLEM

From a Point C in a Right Line given AB , to erect a Right Line CF , at Right Angles.

Take on either side of this Point given two equal Lines, CD , CE , and upon

upon the Right Line DE, erect an Equilateral Triangle; draw the Line FC, and it will be the Perpendicular required.

Demonstration. The Triangles DCF, ECF, have the Side CF common, the Sides DF, EF, equal, the Bases DC, EC, are also equal: Therefore (by the 8th) the Angles DCF, ECF, are equal, (and by the 10th Def.) the Line CF is Perpendicular to AB.

PROPOSITION XII.

PROBLEM.

UPon an indeterminate Right Line BC, from a Point given, that is not in it as A, to let fall a Perpendicular AE.

From the Center A describe a Circle, cutting the Right Line given BC, in the Points G and F, then bisect GF in E, and draw the Right Line AE, which will be the Perpendicular required; draw the Lines AG, AF.

Demonstration. The Triangles GEA, FEA, have the Side AE common, and the Sides EG, EF equal, the Line GF being bisected in E; the Bases AG, AF, being drawn from the Center A, to the Circumference FG, being therefore equal; thence the Angles AEG, AEF, are

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are equal, (by the 8th) and the Line AE
Perpendicular, (by the 10th Def.)

USE.

THis Proposition is not only used by Mathematicians, but almost by all Tradesmen, as Joiners, Carpenters, Shipwrights, Masons, and others: Their Square being a Right Angle; in Fortifications after the French manner, the Right Angled is preferred above all others. We cannot put Surveying in Practice, without making use of Perpendicular Lines: Dialling cannot be without it, in fine, the greatest part of the practice of Tradesmen, supposeth the knowledge of drawing Perpendicular Lines.

PROPOSITION. XII.

THEOREM.

If a Line fall on another Line, it shall make therewith Two Right Angles, or Two Angles equal to two Right Angles.

Let the Line AD fall upon BG: I say that it shall make therewith two Right Angles, or two Angles, the one Obtuse, the other Acute; which being added together, shall be equal to two Right Angles.

Demonstration. Let the Line AD, be Perpendicular to BG, it is evident (by the

the 10th Def.) that the Angles ADH, ADG, are equal, and consequently are Right Angles.

Secondly, let the Line ED be drawn not at Right Angles to BG, draw the Perpendicular AD; (by the 11th) the Angles ADB, ADC, are both Right Angles and are equal to the three Angles ADG, ADE, EDB. Now the Obtuse Angle EDG, and the Acute Angle EDB, are equal to the Three Angles ADG, ADE, EDB: Therefore the Angles EDG, EDB, are equal to Two Right Angles.

COROLLARIES.

1. **H**ence if one Angle ABD be Right, the other ABG is also Right; if one Acute, the other Obtuse, and so on the contrary.

2. If the Line ED falling on BG, maketh the Angle EDB Acute, the Angle EDG shall be Obtuse.

When we know one of the Angles, which is made by a Right Line falling on another Right Line, we know also the other, as for instance, let EDB be 40 degrees, take away 40 degrees from 180, there remains EDB 140. The Practice of this is often of use in Tri-

Trigonometry, and also in Astronomy, to find the excentricity of the Circle, which the Sun describeth in his Annual motion.

PROPOSITION XIV.

THEOREM.

IF Two Right Lines, meeting each other in the same Point in another Line, making therewith Angles equal to Two Right Angles, then shall those Lines make one freight Line.

Let the Lines CA , DA , meet in the same Point A , of the Line AB ; and let the adjacent Angles CAB , BAD , be equal to Two Right Angles. I say, that then the Lines CA , AD , make one freight Line, for if CA be continued, it shall fall precisely on AD . If you deny it, let CA , AE , make one Right Line, and describe a Circle from the Center A .

Demonstration. If you say CAE is a freight Line, the Arch CBE is a Semi-Circle: Now it is also supposed that the Angles CAB , BAD , are equal to Two Right Angles, that is to say, that their measure CBD is a Semi-Circle: Therefore the Arches CBE , CBD , shall be equal, which is impossible, since that the one is a part of the other, there-

therefore the Line CA being continued, shall make one Line with AD.

PROPOSITION XV.
THEOREM.

IF Two Right Lines cut through one another, then are the Two Angles which are opposite equal one to the other.

Let the Lines AB, CD, cut each other in the Point E: I say that the Angles AEC, DEB, which are opposite are equal.

Demonstration. The Line CE falling on the Line AB, maketh the Angles AFC, CEB, equal to Two Right Angles, (*by the 13th Prop.*) in the like manner doth the Line BE falling on the Line CD, make the Angles CEB, BED, equal to Two Right Angles. Therefore the Angles AEC, CEB, taken together, are equal to the Angles CEB, BED, take away the Angle CEB, which is common to both, then (*by the 3d Axiom*) the Angles AEC, DEB, shall be equal.

Corol. If the Two Lines DE, EC, concurring in the same Point E, of the Line AB, making therewith the opposite Angles, AEC, DEB, equal; DE, EC, shall be a streight Line.

Demonstration. The Line EC falling
on

on AB, maketh the Angles AEC, BEC, equal to Two Right Angles (by the 13th.) It is also supposed that the Angle DEB is equal to the Angle AEC. Therefore the Angles DEB, BEC are equal to Two Right, and (by the 14th) the Lines CE, ED, is one streight Line.

U S E.

Use 15.

WE often make use of the two Preceding Propositions, in Catoptricks, to prove that of all the Lines which can be drawn from the Point A, by reflection, to the Point B, those are the shortest which make the Angles of Incidence equal to the Angles of Reflection. For Example, if the Angles BED, AEF, are equal, the Lines AE, EB, are shorter than AF, FB. Draw from the Point B, the Perpendicular BD, and let the Lines BD, DC, be equal: Then draw EC, FC. Now in the first place, in the Triangles BED, CED, the Side DE being common, and the Sides BD, DC, being equal, and the Angles CDE, BDE, being also equal, the Bases BE, CE, shall be equal; as also the Angles BED, DEC, (by the 4th) I might prove by the same manner of Argument, that BF, CF, are equal.

Demonstration. The Angles BED, DEC,

DEC, are equal, it is also supposed that the Angles BED, AEF, are equal: Thence the opposite Angles DEC, AEF, shall be equal; and (by the Cor. of the 15th Prop.) AEC is a straight Line. By consequence AFC is a Triangle, in which the Sides AF, FC, are greater than AE, EC; that is to say AE, EB: For the Lines AE, FC, are equal to AF, EB: Therefore the Lines AF, EB, are greater than AE, BE, and since natural Causes do all by the most shortest Lines; therefore all reflections are made after this sort; that the Angles of Reflection, and of Incidence, are equal.

Moreover, because we can easily prove Prop. XV. that all the Angles which are made upon a Plain, by the meeting of never so many Lines in the same Point, are equal to Four Right Angles; since that in the first Figure of this Proposition the Angles AEC, AED, are equal to two Right, as also BED, BEC; we make this General Rule to determine the number of Polygons, which may be joyned together to Pave a Floor, so we say that four Squares, six Triangles, three Hexigones, may Pave the same; and that it is for this reason the Bees make their cellules Hexagonal.

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PROPOSITION XVI.
THEOREM.

The exterior Angle of a Triangle is greater than either of the inward and Opposite Angles.

Continue the Side BC of the Triangle ABC: I say that the exterior Angle ACD, is greater than the interior opposite Angle ABC or BAC: Imagine that the Triangle ABC moveth along BD, and that it is Transported to CED.

Demonstration. It is impossible that the Triangle ABC should be thus moved, and that the Point A should not alter its place moving towards E: Now if it moveth towards E, the Angle ECD, that is to say ABC, is less than the Angle ACD; therefore the Interior Angle ABC, is less than the Exterior ACD.

It is easy to prove that the Angle A, is also less than the Exterior ACD: For having continued the Side AC to F, the opposite Angles BCF, ACD, are equal (by the 15th) and making the Triangle ABC to slide along the Line ACF, it is demonstrable, that the Angle BCF, is greater than the Angle A.

USE.

U S E.

WE draw from this Proposition several very useful conclusions. The First, that from a given Point there cannot be drawn no more than one Perpendicular to the same Line. Example, Let the Line AB be Perpendicular to the Line BC : I say AC shall not be Perpendicular thereto, because the Right Angle ABC , shall be greater than the Interior ACB : Therefore ACB shall not be a Right Angle, neither AC a Perpendicular.

Secondly, that from the same Point A , there can only be drawn two equal Lines; for Example, AC , AD , and that if you draw a Third AE , it shall not be equal. For since AC , AD , are equal, the Angles ACD , ADC , are equal, (by the 5th.) now in the Triangle AEC , the Exterior Angle ACB is greater than the Interior AEC ; and thus is the Angle ADE greater than AED . Therefore the Line AE , greater than AD ; and by consequence AC , AE , are not equal.

Thirdly, if the Line AC maketh the Angle ACB acute, and ACF obtuse, the Perpendicular drawn from A shall fall on the same Side which the Acute Angle is of; for if one should say that AE is

a Perpendicular, and the Angle AEF is Right; then the Right Angle AEF would be greater than the Obtuse Angle AEF. These conclusions we make use of to measure all Parallelograms, Triangles, and Trapeziums, and so Reduce them into Rectangular Figures.

PROPOSITION XVII.

THEOREM.

TWO Angles of any Triangle, are less than Two Right Angles.

Let ABC be the proposed Triangle; I say that Two of the Angles taken together BAC, BCA, are less than Two Right Angles. Continue the side CA, to D.

Demonstration. The Interior Angle C, is less than the Exterior BAD, (by the 16th.) Add to both the Angle BAC, the Angles BAC, BCA, shall be less than the Angles BAC, BAD; which latter are equal to Two Right (by the 13th.) Therefore the Angles BAC, BCA, are less than Two Right.

I might Demonstrate after the same manner, that the Angles ABC, ACB, are less than Two Right, by continuing the Side BC.

Coroll. If one Angle of a Triangle be either Right or Obtuse, the other Two shall be Acute. This

This Proposition is necessary to demonstrate those which follow.

PROPOSITION XVIII.
THEOREM.

THE greatest Side of every Triangle, subtends the greatest Angle.

Let the Side BC, of the Triangle ABC, be greater than the Side AC. I say, that the Angle BAC, which is opposite to the Side BC, is greater than the Angle B, which is opposite to the Side AC. Cut off from BC, the Line CD, equal to AC and draw AD.

Demonstration. Seeing that the Sides AC, CD, are equal, the Triangle ACB is an Isosceles Triangle, (by the 5th) the Angles CDA, CAD, are therefore equal. Now the whole Angle BAC is greater than the Angle CAD: Thence the Angle BAC is greater than the Angle CDA, which being exterior, in regard of the Triangle ABD, is greater than the interior B (by the 16th.) Therefore the Angle BAC is greater than the Angle B.

PROPOSITION XIX.
THEOREM.

IN every Triangle, the greatest Angle is opposite to the greatest Side.

Let the Angle A of the Triangle BAC, be greater than the Angle ABC. I say that the Side BC, opposite to the Angle A, is greater than the Side AC, opposite to the Angle B.

gle A, is greater than the Side AC, opposite to the Angle B.

Demonstration. If the Side BC, was equal to the Side AC; in this case the Angles A and B would be equal (*by the 5th*) which is contrary to the Hypothesis: If the Side BC was less than AC, then the Angle B would be greater than A, which is also contrary to the Hypothesis. Wherefore I conclude that the Side BC is greater than AC.

U S E.

Use 19.

WE prove by these Propositions, not only that from the same Point to a Line given, there can be but one Perpendicular drawn, but also that that Perpendicular is the shortest Line of all those Lines which might be drawn to the said Line. As for instance, if the Line RV be Perpendicular to ST, it shall be shorter than RS: because the Angle RV S being Right, the Angle RSV shall be Acute (*by the Cor. of the 17th*) and the Line RV shall be shorter than RS (*by the Preceding*.) For this Reason Geometricians always make use of the Perpendiculars in their measuring; and Reducing irregular Figures into those whose Angles are Right.

I further add, that seeing there can
only

only be drawn Three Perpendiculars to one and the same Point, it cannot be imagined that there are more than Three Species of quantity; viz. A Line, a Surface, and a Solid.

We also prove by these Propositions, that a Boul which is exactly round, being put on a Plain, cannot stand but on one determinate Point. As for Example, Let the Line AB Represent the Plain, and from the Center of the Earth C, let the Line CA be drawn Perpendicular to the Line AB. I say that a Boul being placed on the Point B, ought not to stand on that Point, because no heavy Body will stand still while it may Descend. Now the Boul B going towards A, is always descending, and cometh nearer and nearer the Center of the Earth C. Because in the Triangle CAB, the Perpendicular CA is shorter than BC.

We also prove in like manner, that a liquid Body ought to Descend from B to A, and that the Surface ought to be round.

PROPOSITION XX.

THEOREM.

The Two Sides of a Triangle taken together are greater than the Third.
I say that the Two Sides TL, LV,
C 2 of

of the Triangle TLV, are greater than the Side TV. Some prove this Proposition by the Definition of a straight Line, which is the shortest which can be drawn between Two Points: Therefore the Line TV is less than the Two Lines TL, LV.

But we may demonstrate the same another way, continue the Side LV to R, and let LR, LT, be equal, then draw the Line RT.

Demonstration. The Sides LT, LR, of the Triangle LTR, are equal. Therefore the Angles R, and RTL equal (by the 5th) but the Angle RTV, is greater than the Angle RTL: Therefore the Angle RTV, is greater than the Angle R, and (by the 17th) in the Triangle RTV, the Side RV, that is to say, the sum of the Sides LT, LV, are greater than the Side TV.

PROPOSITION XXI.

THEOREM

IF on the same Base you draw a lesser Triangle in a greater, the Sides of the lesser shall be shorter than the greater, but contain a greater Angle.

Let the less Triangle ADB be drawn within the greater ACB, on the same Base AB. I say in the first place that the

the Sides AC, BC, are greater than the Sides AD, BD. Continue the Side AD unto E.

Demonstration. In the Triangle ACE the Sides, AC, CE, taken together, are greater than the Side AE, (by the 20th.) Add thereto the Side EB, the Sides AC, CEB, shall be greater than the Sides AE, EB. Likewise in the Triangle DBE, the Two Sides BE, ED, taken together, are greater than BD, and adding thereto AD, the Sides AE, EB, shall be greater than AD, BD.

Moreover, I say, that the Angle ADB, is greater than the Angle ACB: For the Angle ADB is exterior in respect of the Triangle DEB, it is therefore greater than the Interior DEB (by the 16th.) Likewise the Angle DEB being exterior in respect of the Triangle ACE, is greater than the Angle ACE: Therefore the Angle ADB, is greater than the Angle ACB.

USE

WE Demonstrate in Opticks by this Prop. 21. Proposition, that if from the Point C, one should see the Base AB, it would seem less than if one should see the same from the Point D, according to this Principle, that quantities seen

Q3

C 3

under

under a greater Angle, appear greater, for which reason Vitruvius would that the Tops of very high Pillars should be made but little tapering, because that their Tops being at a good distance from the Eyes, will of themselves appear very much diminished.

PROPOSITION XXII.

PROBLEM.

TO Make a Triangle, having its Sides equal to Three Right Lines given, provided that Two of them be greater than the Third.

Let it be proposed to make a Triangle, having its Three Sides equal to the Three given Lines, AB, D, E, take with your Compasses the Line D, and putting one Foot thereof in the Point B, make an Arch: Then take in your Compasses the Line E, and putting one Foot in the Point A, cross with the other Foot, the former Arch in C, draw the Lines AC, BC. I say, that the Triangle ABC, is what you desire.

Demonstration. The Side AC Is equal to the Line E, since it is the Radius of an Arch drawn on the Center A, equal in Length to the Line E, likewise the Side BC is equal to the Line D: Therefore the Three Sides AC, BC,

BC, AD, are equal to the Lines E, D, AB.

U S E.

WE make use of this Proposition to make a Figure equal, or like unto another, for having divided the Figure into Triangles, and making other Triangles, having their Sides equal to the sides of the Triangles in the proposed Figure, we shall have a Figure like unto the same in all respects: But if we desire it should be only like thereunto, but lesser, for Example; if we would have the Form of any Plain or Piece of Ground on Paper; having divided the same into Triangles, and measured all their Sides, we make Triangles like unto those of the Plain, by the help of a Scale of equal parts, from which we take the number of Parts, which their Sides contain, whether Feet, Rods, or any other Measure, and applying them as is here taught.

PROPOSITION XXIII.

PROBLEM.

TO make an Angle equal to an Angle given in any Point of a Line.

Let it be proposed to make an Angle equal to EDF, at the Point A, of

the Line AB, at the Points A and D, as Centers, draw two Arches BC, EF, with the same extent of the Compasses; then take the Distance EF between your Compasses; put one Foot in B, and cut off BC, and draw AC. I say, that the Angles BAC, EDF, are equal.

Demoustration. The Triangles ABC, DEF, have the Sides AB, AC, equal to the Sides DE, DF; since that the Arches BC, EF, were described with the same extent of the Compass, they have also their Bases BC, EF, equal: Therefore the Angles BAC, DEF, are equal (*by the 8th.*)

U S E.

This Problem is so necessary in Surveying Fortifications, Prospective, Dialling, and in all other parts of the Mathematicks; so that the greatest part of their Practice would be impossible, if one Angle could not be made equal to another, or of any number of Degrees required.

PROPOSITION XXIV.

THEOREM.

IF Two Triangles which have Two Sides of the one equal to the Two Sides

Sides of the other, that which, hath the greatest Angle hath the greatest Base.

THEOREM

Let the Sides AB , DE , AC , DF , of the Triangles ABC , DEF , be equal, and let the Angle BAC , be greater than the Angle EDF . I say, that the Base BC , is greater than the Base EF . Make the Angle EDG , equal to the Angle BAC , (by the 23d.) and the Line DG equal to AC , then draw EG . In the first place the Triangles ABC , DEG , having the Sides AB , DE , AC , DG equal, and the Angle EDG , equal to BAC , their Bases BC , EG , are equal (by the 4th) and the Lines DG , DF , being equal to AC , they shall be equal amongst themselves.

Demonstration. In the Triangle DGF , the Sides DG , DF , being equal, the Angles DGF , DFG , are equal (by the 5th) but the Angle EGF , is less than DGF , and the Angle EFG is greater than DFG . Therefore in the Triangle EFG , the Angle EFG , shall be greater than the Angle EGF , thence (by the 18th) the Line EG , opposite to the greatest Angle EFG , shall be greater than EF , thence BC , equal to EG , is greater than the Base EF .

PROPOSITION XXV.

THEOREM.

OF Two Triangles which have Two Sides of the one equal to Two Sides of the other, that Triangle which hath the greatest Base hath also the greatest Angle.

Let the Sides AB , DE , AC , DF , of the Triangles ABC , DEF , be equal, and let the Base BC be greater than the Base EF . I say, that the Angle A shall be greater than the Angle D .

Demonstration. If the Angle A were not greater than the Angle D , it would be equal, or less; if equal, in this case the Bases BC would be equal (by the 4th.) If less, then the Base EF would be greater than the Base BC , (by the 24th) both which is contrary to our Hyp.

These Propositions are of use to Demonstrate those which follow.

PROPOSITION XXVI.

THEOREM.

A Triangle which hath One Side and Two Angles equal to those of another, shall be equal thereto in every respect.

Let

Let the Angles ACB , DEF , ACD , DFE , of the Triangles ABC , DEF , be equal; and let the Sides BC , EF , which are between those Angles, be also equal to each other. I say, that their other Sides are equal. For Example, AC , DF ; but let it be imagined that the Side DF is greater than AC , and that GF is equal to AC ; and draw the Line GE .

Demonstration. The Triangles ABC , GEF , have the Sides EF , BC , AC , GF , equal; the Angle C is also supposed equal to the Angle F , thence (by the 4th) the Triangles ABC , GEF , are equal in every respect, and the Angles GEF , ABC are equal; but according to our first Hyp. the Angles ABC , DEF , were equal; by this Argument the Angles DEF , GEF , would be equal; that is to say, the whole equal to a part, which is impossible; therefore DF cannot be greater than AC , nor AC greater than DF ; because the same Demonstration might be made in the Triangle ABC .

Secondly, Let us suppose that the Angles A and D , C and F , are equal; and that the Sides BC , EF , which are opposite to the equal Angle A and D are also equal to each other. I say, the

other Sides are equal; for if DF be greater than AC , make GF equal to AC , and draw the Line GE .

Demonstration. The Triangles ABC , GEF , have the Sides EF , BC , FG , CA equal, they are then equal in every respect (by the 4th) and the Angles EGF , BAC , shall be equal, but according to our Hyp. A and D were equal, thus the Angles D , and EGF , should be equal, which is impossible, since that the Angle EGF being exterior, in respect of the Triangle EGD , it ought to be greater than the Interior Angle D , (by the 16th.) Therefore the Side DF , is not greater than AC .

U S E

Mc-26.

THales made use of this Proposition to measure inaccessible Distances, the Distance AD being proposed from the Point A , he draws the Line AC Perpendicular to AD ; then, placing a Semicircle at the Point C , he measureth the Angle ACD ; then he takes an Angle equal thereto on the other side, drawing the Line CB , until it meets the Line DA , continued to the Point B . He Demonstrates that the Lines AD , AB , were equal; so measuring actually the accessible Line, he might know

known by that means the other. For the Two Triangles ADG , ABC , have the Right Angles CAD , CAB , equal; both the Angles ACD , ACE , were taken equal to each other; and the Side AC is common to both Triangles. Therefore (by the 26th) the Sides AD , AB , are equal.

LEMMA I

A Line which is Perpendicular to one Parallel, is also Perpendicular to the other. Lem 26.

Let the Lines AB , CD , be Parallel to each other, and let EF be Perpendicular to CD . I say that it is Perpendicular to AB . Cut off two equal Lines CE , ED , at the Points C and D erect two Perpendiculars to the Line CD , which shall also be equal to FE , by the Definition of Parallels; and draw the Lines EC , ED .

Demonstration. The Triangles CEF , FED , have the Side FE common, the Sides EC , ED , are equal, the Angles at F are Right, and by consequence equal. Therefore (by the 4th) the Bases EC , ED , the Angles FED , FEC , FDE , FCE , are equal; and those two last being taken away from the Right Angles ACE , BDE , leaveth the equal Angles EDB , ECA . Now the Triangles CAE , DBE , shall (by the 4th) have the Angles

gles DEB , CEA , equal; which Angles being added to the equal Angles CEF , FED , maketh equal Angles FEB , FEA . Therefore EF is Perpendicular to AB .

PROPOSITION XXVII.

THEOREM.

IF a right Line falling upon two Right Lines, make the Alternate Angles equal, the one of the other, then are the Right Lines Parallel.

Let the Line EH fall on the Right Lines AB , CD , making therewith, the Alternate Angles AFG , FGD , equal. I say in the first place, that the Lines AB , CD , shall not meet, although continued as far as one lists. For supposing they should meet in I , and that FBI , CDI are two straight Lines.

Demonstration. If FBI , GDI , be two straight Lines, then FIG is a Triangle; then (by the 16th) the exterior Angle AFG shall be greater than the interior FGI . Wherefore that the equality of the Angles may subsist, the Lines AB , CD , must never meet each other.

But because we have Examples of some crooked Lines that never intersect, which notwithstanding are not Parallels, but approach continually.

To prove the foregoing, I make another Demonstration, as followeth. First,

I say that if the Line EF falleth on the Lines AB , CD , maketh the Alternate Angles AFG , FGD , equal; the Lines AB , CD , are Parallels, that is in every part equidistant from each other; for which reason the Perpendiculars shall be equal to each other. Draw from G , to the Line AB , the Perpendicular GA , and CD being taken equal to AF , draw FD .

Demonstration. The Triangles AGF , FGD , have the Side GF common, the Side GD was taken equal to AF ; it is supposed that the Angles AFG , FGD , are equal. Therefore (by the 4th) AG , FD , are equal, and the Angle GDF is equal to the Right Angle GAF . Thence FD is Perpendicular. Furthermore that AB is Parallel to CD ; for the Parallel to CD , is to be drawn from the Point F , and must pass through the Point A , according to our Definition of Parallels, which is that the Perpendiculars AG , FD , are equal.

PROPOSITION XXVIII.

THEOREM.

If a Right Line falling upon two Right Lines, make the Exterior Angle equal

equal to the Interior opposite Angle of the other on the same Side, and the Two Interior on the same Side equal to two Right, then are the Right Lines Parallel.

Having drawn a Figure like unto the former, let the Line EH fall on AB, CD, make in the first place the Exterior Angle EFB, equal to the Interior opposite Angle of the other on the same Side FGD. I say that the Lines AB, CD, are Parallel.

Demonstration. The Angle EFB, is equal to the Angle AFG, (by the 15th) and it being supposed that EFB is also equal to FGD. Thence the Alternate Angles AFG, FGD, shall be equal, and (by the 27th) the Lines AB, CD, are Parallel.

Secondly, I say that if the Angles BFG, FGD, which are Interior on the same Side, be found equal to Two Right Angles, the Lines AB, CD, shall be Parallel.

Demonstration. The Angles AFG, BFG, are equal to two Right (by the 13th) and it is supposed that the Angles GFB, FGD, are equal to two Right; therefore the Angles AFG, FGD, are equal. And (by the 27th) the Lines AB, CD, shall be Parallel.

PRO-

PROPOSITION XXIX.

THEOREM.

IF a Right Line fall upon Two Parallels, the Alternate Angles shall be equal: The Exterior Angle shall be equal to the Interior and Opposite; and the Two Interior Angles on the same Side shall be equal to Two Right Angles.

Let the right Line EH fall on two Parallels AB, CD. I say in the first place that the Alternate Angles AFG, FGD, are equal. Draw from the Points F and G the Perpendiculars GA, GD, which by the Definition of Parallels are equal.

Demonstra. I the rectangled Triangles AFG, FGD, the Sides FD, AG, being equal, as also the Right Angles A and D; and the Side FG being common: I say in the first place that the Side GD is equal to AF. For if GD were greater; then having cut off the Line DI equal to AF, and draw FI; then the Triangles AFG, FDI, would have their Bases GF, FI, equal, which cannot be, since that the Angle D being Right, the Angle FID is Acute, and FIG Obtuse (by the 13th) and (by the 18th) in the Triangle FIG, the Side FG opposite to the Obtuse Angle FIG, is

is greater than FI. Wherefore DG equal to AF; and the Triangles AFG, FGD, having all the Sides equal, the alternate Angles AFG, FGD, equal, because opposite to the equal Sides AG, FD.

Again, I say that the Exterior Angle EFB is equal to the Interior FGD; because (by the 15th) it is equal to its opposite AFG, which is equal to his Alternate FGD.

Now, since the Angles AFG and GFB are equal to two Right; take away AFG, and in the place thereof substitute his Alternate FGD, the Interior Angles GFB, FGD, shall be equal to Two Right Angles.

U S E.

Use 29.

ERatosthenes applyeth those Propositions to the measuring of the Circumference of the Earth. He supposeth that two Rays of Light proceeding from the Center of the Sun to two Points on the Earth, are Physically Parallel. He supposeth also that at Syena a Town in Egypt, the Sun was in the Zenith on the day of the Solstice, because that their Cisterns, Wells, and such like, were enlightened by the Suns Rays to the very bottom, he measured also the distance from

from Alexandria to Syena. To demonstrate which, let us suppose Alexandria to be situated at the Point B, and Syena at the Point A, where let the Style BC be erected Perpendicular to the Horizon, and let DE, EC, be two Rays proceeding from the Center of the Sun, when in the Solstice, which are Parallels between themselves; let DA that Ray of Light Perpendicular to Syena, be supposed to be continued to E, the Center of the Earth.

Now having observed at Alexandria the Angle GCB, which is made or included between the Perpendicular CB, and the Ray of Light EC: I say that because the aforesaid Rays are Parallel, the Alternate Angles GCB, SPA, are equal. We know then the angle AFB, and the measure of AB the distance of Alexandria to Syena taken in Degrees. Now supposing this Distance be known in Miles, the Circumference of the Earth shall be easily found by a simple Rule of Three: For if 90 Degrees give to many Miles, what shall 360

PROPOSITION XXX.

THEOREM.

Right Lines parallel and the Angles, are also parallel to the other. Let

Let the Lines AB, FE, be parallel to the Line CD; I say they are parallel the one to the other. Let the Line GK cut all three.

Demonstration. Seeing that the Lines AB, CD are parallel; the alternate Angles AHL, HID, shall be equal (*by the 29th*) and because the Lines CD, FE, are also parallel; the Exterior Angle HID shall be equal to the Interior ILE (*by the same*). Therefore the alternate Angles AHL, ILE, shall be equal, and the Line AB, FE, Parallels (*by the 28th*.)

PROPOSITION XXXI.

PROBLEM.

TO Draw a Line Parallel to another, through a Point given.

It is proposed to draw a Line through the Point G, which shall be parallel to the Line AB. Draw the Line CE, and make the Angle ECD, equal to the Angle CEA: I say that the Line CD, is Parallel to AB.

Demonstration. The alternate Angles DCE, CEA, are equal: Therefore the Lines CD, AB, are Parallel.

One might easily demonstrate the eleventh Maxim, that is to say, that if a Line falling on two other Lines, maketh the Interior Angles less than two Right,

Right, those two Lines shall intersect.

Let the Line AC falling on the Lines Ax. 11.

AB, CD, make the Interior Angles ACD, CAB, less than two Right: I say the Lines AB, CD, shall Intersect. Let the Angles ACD, CAE, be equal to Two Right, the Lines AE, CD, are Parallels (by the 29th.) Take at discretion the Line AB, and through the Point B draw the Line EF Parallel to CA. Then take EB as many times as shall be necessary to fall below the Line CD, as in this Figure I have taken it but twice, wherefore EB, BF, are equal. Draw from the Point F a Parallel FG, equal to AE, and join GE. I say that the Line ABG, is but one straight Line; and that so the Line AB concurring with FG, if CD be continued, since it cannot cut the Parallel FG, it shall cut the Line BG between B and G. to Qd shd shd rll

Demonstration, The Triangles AEB, BFG, have the Sides AE, FG, BE, BF, equal, as also all the Alternate Angles AEB, BFG, (by the 29th.) Therefore they are equal in every respect (by the 4th) and the opposite Angles ABE, FBG, are equal; and consequently (according to the 15th) AB, BG, maketh one straight Line. (Qd shd shd rll)

U S E.

USE.

The use of Parallel Lines is very necessary, first in Perspective, since that the Appearances or Images of the Parallel Lines on the Draft are Parallel between themselves. In Navigation, like Runns in our Charts, are Parallel to each other. Likewise the Hour Lines on Polar Dials, the Compass of Proportion or Sector is grounded also on Parallel Lines.

PROPOSITION XXXII.

THEOREM.

The Exterior Angle of a Triangle is equal to the two Interior opposite Angles taken together, and the Three Angles of a Triangle are equal to Two Right.

Let the Side BC of the Triangle ABC be continued to D. I say that the Exterior Angle ACD is equal to the two Interior Angles A, and B, taken together. Draw from the Point C the Line CE, Parallel to AB.

Demonstration. The Lines AB, CE, are Parallels; therefore (by the 29th) the Alternate Angles ECA, CAB, are equal; and (by the same) the Exterior Angle ECD is equal to the Interior B.

By

By consequence the whole Angle ACD, is equal to the two Angles ACE, ECD, of whom it is composed, it shall then be equal to A and B taken together.

In the second place, the Angles ACD, ACB, are equal to two Right (by the 17th.) And I have demonstrated that the Angle ACD was equal to the Angles A and B taken together: Therefore the Angle ACD, is equal to A and B, that is to say all the Angles of the Triangle ABC, are equal to Two Right, or 180 degrees.

Corollary 1. The three Angles of one Triangle, are equal to the three Angles of another Triangle.

Coroll. 2. If two Angles of a Triangle be equal to two Angles of another Triangle; the remaining Angle in the one shall be equal to the remaining Angle in the other.

Coroll. 3. If a Triangle have one of its Angles Right, the other Two shall be Acute; and being taken together, shall be equal to one Right Angle.

Coroll. 4. From one and the same Point of a Line there can be drawn but one Perpendicular; because a Triangle cannot have Two Right Angles.

Coroll. 5. The Perpendicular is the shortest Line which can be drawn from a Point to a Line.

Corol.

Coroll. 6. In a Right Angled Triangle, the greatest Angle is a Right Angle, and the longest side is opposite thereto.

Coroll. 7. Each Angle of an Equilateral Triangle containeth 60 degrees, that is to say, the third of 180.

U S E

Uſe 32.

THis Proposition is of use to us in Astronomy, to determine the Parallax. Let the Point A represent the Center of the Earth; and from the Point B on the Superficies of the Earth, let there be taken by Observation the Angle DBC, this is to say, the apparent Distance of a Planet or Comet from the Zenith D. I say, if the Earth were transparent, this Planet or Comet viewed from the Center of the Earth A, would appear distant from the Zenith D, equal to the quantity of the Angle CAD, which is less than the Angle CBD. For the Angle CBD being Exterior in respect of the Triangle ABC, is equal (by the 32d) to the opposite Angles A and C. Whence the Angle C shall be equal to the excess of the Angle CBD, above the Angle A. From whence I conclude, that if I know by an Astronomical Table, how far that Planet

Planet or Comet ought to appear Distant from the Zenith to one which should be in the Center of the Earth, and if I observe at the same time, the difference of those Two Angles shall be the Parallax, viz. The Angle BCA.

PROPOSITION XXXIII.

THEOREM.

IF Two equal and Parallel Lines be joyned together with Two other Right Lines, then are those Lines also equal and Parallel.

Let the Lines AB, GD, be Parallel and equal, and let them be joyned with AG, BD. I say that the Lines AG, BD, are equal and Parallel, draw the Diagonal BG.

Demonstration. Seeing the Lines AB, GD, are parallel, the Alternate Angles ABG, BGD, shall be equal (by the 29th) and the Side GB being common to both the Triangles ABG, BGD, and the Sides AB, GD, equal, with the Angles ABG, BGD, equal, as before, the Bases of those Triangles AG, BD, shall be equal (by the 4th) as also the Angles DBG, BGA; which because Alternate, the Lines AG, BD, shall be Parallel (by the 27th.)

D

USE.

USE.

Use 33.

THis Proposition is put in practise to measure as well the Perpendicular height AG of Mountains as their Horizontal Lines C, G, which are hidden in their thickness. To effect which, we make use of a very large square ABD, putting one end thereof in A, in such manner that the other Side thereof BD may be Perpendicular to the Horizon, then we measure the Sides AD, BD, then we do the like again at the Point B, and measure BE, EC, the Sides Parallel to the Horizon, that is to say AD, BE, being added together, gives the Horizontal Line CG, and the Perpendicular Sides BD, EC, being added gives the Perpendicular Height AG.

PROPOSITION XXXIV.

THEOREM.

THe Sides, and the opposite Angle in a Parallelogram are equal, and the Diameter doth divide the same into Two equal parts.

Let the Figure AB, CD, be a Parallelogram, that is to say, let the Sides AB, CD; AC, BD, be Parallel. I say that the opposite Sides AB, CD; AC, BD, are equal; as also the Angles A and

and D; ABD, ACD; and that the Diameter BC doth divide the Figure into Two equal parts.

Demonstration. The Lines AB, CD, are supposed Parallel: Therefore the Alternate Angles ABC, BCD, shall be equal (*by the 29th.*) Likewise the Sides AC, BD, being supposed Parallel; the Alternate Angles ACB, CBD, are equal. Both which Triangles have the same Side BC, and the Angles ABC, BCD; ACB, CBD, equal, they shall be equal in every respect, (*by the 26th.*) Therefore the Sides AB, CD; AC, BD; and the Angles A and D are equal; and the Diameter divides the Figure into Two equal parts; and seeing the Angles ABC, BCD; ACB, CBD, are equal, adding together ABC, CBD; BCD, ACD, we conclude that the opposite Angles ABD, ABC, shall be equal.

USE.

Surveyers have sometimes occasion to use 34. make use of this Proposition, to part or divide Lands. If the Field be a Parallelogram, it may be divided into Two equal parts by the Diameter AD. But if one be obliged to divide the same into Two equal parts from the Point E; divide the

Diameter into two equal parts in the Point F, and draw the Line EFG, that Line shall divide the Field into Two equal parts; for the Triangle AEF, GFD, which have their Alternate Angles EAF, FDG; AEF, FGD; and the Sides AF, FD, equal; are equal (by the 26th.) And since the Trapezium BEFD, together with the Triangle AFE, that is to say, the Triangle ADB, is one half of the Parallelogram (by the 34th) the same Trapezium EFBD, together with the Triangle DEG, shall be one half of the Figure or Field, and the Line EG divideth the same into Two equal parts.

PROPOSITION XXXV.

THe Parallelograms are equal, when having the same Base, they are between the same Parallel Lines.

Let the Parallelograms ABEC, ABDF, have the same Base AB, and be between the same Parallels AB, CD: I say they are equal.

Demonstration. The Sides AB, CE, are equal (by the 34th,) also AB, FD; wherefore CE, FD, are equal; and adding EF to each, the Lines CF, ED, shall be equal; the Triangles CFA, EBD, have the Sides CA, EB; CF, ED, equal with the Angles DEB, FCA,
(by

(by the 29th) the one being Exteriour, and the other Interiour on the same Side, whence (by the 4th,) the Triangles ACE, BED, are equal; and taking away from both that which is common to both, that is to say the little Triangle EGF; the Trapezium FGBD, shall be equal to the Trapezium CAGE; and adding to both the little Triangle AGB, the Parallelograms ABEC, ABDF, shall be equal.

Demonstration by the method
of Indivisibles.

THis method is new invented by Cavalerius, which is approved of by some, but rejected by others. It consisteth in this imagination or supposition, That Surfaces are Composed of Lines like so many Threads. Now it is certain, that two pieces of Cloth shall be equal, if in both there be found the same number of Threads equal in length; and that they are as close joyned or woven in the one, as in the other.

Let there be proposed two Parallelo-grams ABEC, ABDE, on the same Base AB, and between the same Parallels AB, CD. Moreover draw in the Parallelogram ABEC, as many Lines as you please, which let be continued into the
D 3 other

other Parallelogram, it is evident, that there will be no more Lines in the one than in the other; and that they shall be equal in length, that is to say, to the Base AB; neither shall they be more close joyned in the one, than in the other; wherefore the Parallelograms are equal.

PROPOSITION XXXVI.

THEOREM.

THe Parallelograms are equal, which being between the same Parallels have equal Bases.

Let the Bases CB, OD, of the Parallelograms ACBF, ODEG, be equal, and be both between the Parallels AE, CL. I say that those Parallelograms are equal; draw the Lines CG, BE.

Demonstration. The Bases CB, OD, are equal: OD, GE are also equal: Whence CB, GE, are equal and Parallel; and by consequence (according to the 33d.) CG, BE, shall be equal and Parallel; and CBEG shall be a Parallelogram equal to CBFA (by the 35th) because they have the same Bases. In like manner taking GE for Base, the Parallelograms GODE, CBEG, are equal (by the same.) Wherefore the Parallelograms ACBF, ODEG, are equal.

USE.

USE of the foregoing.

WE Reduce Oblique Angled Pa- Use 35.
rallelograms, as $CBE G$, or
ODEG, into Rectangled Parallelograms;
as $CBFA$, whereby, by measuring this
last, which is easily done thus; Multi-
ply AC , by CB , the Product shall be e-
qual to the Parallelogram $ACBF$, and
consequently to the Parallelograms
 $CBEG$, or $ODEG$.

PROPOSITION XXXVII.

THEOREM.

THE Triangles are equal, which
having the same Base, are between
the same Parallels.

The Triangles ACD , CDE , shall
be equal, if they have the same Base
 CD , and if they are included between
the Parallels AF , CH . Draw the
Lines DB , DF , Parallel to the Lines
 AC , CE , and you will have made two
Parallelograms.

Demonstration. The Parallelograms
 $ACDB$, $ECDF$, are equal (by the 36th.)
The Triangles ACD , CDE , are equal
to the one half of those Parallelograms
(by the 34th.) Therefore the Triangles
 ACD , CDE , are equal.

PROPOSITION XXXVIII.
THEOREM.

THE Triangles are equal, which having equal Bases are between the same Parallels.

The Triangles ACD, GEH, are equal if they have their Bases CD, GH, equal, and if they be included between the same Parallels AF, CH. Draw the Lines BD, HF, Parallel to the Sides AC, EG; and you shall have made two Parallelograms.

Demonstration. The Parallelograms ACDB, EGHF, are equal (*by the 36th*) the Triangles ACD, EGH, are the one half of those Parallelograms (*by the 34th.*) Thence by consequence they are equal to each other.

U S E.

Uſe 38.

IN the two laſt Propoſitions we have the practical way of dividing a Triangular Field into two equal parts; for example ABC. Divide the Line BC, which let be taken for the Baſe, into Two equal parts in D: I ſay, the Triangles ABD, ADC, are equal. For if you imagine a Line Parallel to BC, paſſing by the Point A, thoſe Triangles ſhall have equal Baſes, and ſhall be between

tween the same Parallels, and by consequence equal. We might make other Divisions grounded on the same Proposition, which I omit for brevity sake.

PROPOSITION XXXIX.

PROBLEM.

Equal Triangles standing on the same Base, are between the same Parallels.

If the Triangles ABC, DBC, which have the same Base BC, are equal; the Line AD drawn through the tops of those Triangles, shall be Parallel to the Base. For if AD, BC, are not Parallels; having drawn a Parallel through the Point A, the same shall fall either under AD, as AO; or above as AE. Let us suppose it falleth above, continue BD, untill it meets AE in the Point E; then draw the Line CE.

Demonstration. The Triangles ABC, EBC, are equal (by the 38th,) seeing the Lines AE, BC, are Parallel; it is also supposed that the Triangles ABC, DBC, are equal; thence the Triangles DBC, EBC, should be equal; which is impossible; the first being a part of the second. From whence I conclude, that the Parallel to BC cannot be drawn above as AE.

I further say, that it cannot fall under AD, as AO, because then the Triangle BOC should be equal to ABC, by consequence to the Triangle DBC, that is to say, a part equal to the whole. It must then be granted that the Line AD is Parallel to BC.

PROPOSITION XL.

THEOREM.

Equal Triangles standing upon equal Bases, taken on the same Line, are betwixt the same Parallels.

If the equal Triangles ABC, DEF, have their Bases AB, DE, equal, and taken on the same Line AE, the Line CF drawn through the Vertices of those Triangles, shall be Parallel to AE; for if it be not Parallel, having drawn from the Point C a Parallel Line to AE, that Line shall fall above CF, as CG, or underneath as CL.

Demonstration. If it passeth above CF, as CG, continue DF until it meet CG in G, and draw EG; the Triangles ABC, DGE, would then be equal (*by the 38th.*) and having supposed that ABC, DEF, are equal, DFE, DGE, should be also equal; which cannot be true, the one being a part of the other; therefore the Parallel passeth not above CF.

CF. I further add, that it cannot pass underneath as CI, because the Triangles ABC, DEI, would be equal to each other, and by consequence DEI, DEE, the part and the whole would be equal, which is absurd. Therefore CF is the only Line which is Parallel to AE.

PROPOSITION XLI.

THEOREM.

IF a Parallelogram have the same Base with a Triangle, and be between the same Parallels, then is the Parallelogram double to the Triangle.

If the Parallelogram ABCD, and the Triangle EBC have the same Base BC, and between the same Parallels AE, BC, the Parallelogram shall be double to the Triangle, draw the Line AC.

Demonstration. The Triangles ABC, BCE, are equal (*by the 28th.*) Now the Parallelogram ABCD is double to the Triangle ABC, (*by the 24th.*) it is therefore double to the Triangle BCE.

USE.

THe general method of measuring of Use 391
the Area or Superficies of a Triangle, is grounded on this Proposition. Let the Triangle ABC be proposed, we draw from the Angle A the Line AD Perpendicular

dicular to the Base BC ; and Multiplying the Perpendicular AD by the Semi-Base BE , the Product gives the Area of the Triangle; because Multiplying AD or EF by BE , we have the Content of the Rectangle $BEFH$, which is equal to the Triangle ABC . For the Triangle ABC is half of the Rectangle $HBCG$, (by the 41st.) as well as $BEFH$.

We 39.

We measure all sorts of Right Lined Figures, as $ABCDE$; Reducing them first into Triangles BCD , ABD , AED ; by drawing the Lines AD , BD , and the Perpendiculars CG , BF , EL . For Multiplying the half of BD by CG , and the half of AD by BF , and by EL , we have the Area of all these Triangles; which added together is equal to the content of the Right Lined Figure $ABCDE$.

We 39.

We find the Area of Regular Polygons, by Multiplying one half of their Peripheries by the Perpendicular drawn from their Centers to one of their Sides; for Multiplying IG by AG , we have the Rectangle $HKLM$ equal to the Triangle AIB : And doing the same for every Triangle, taking always the Semi-Base, we have the Rectangle $HKON$, whose Side KO Composed of the Semi-Bases, and consequently equal to the Semi-Periphery; and the Side HK equal to the Perpendicular IG .

AC-

According to this Principle, Archimedes hath Demonstrated, that a Circle is equal to a Rectangle comprehended under the Semi-Diameter, and a Line equal to the Semi-Circumference.

PROPOSITION XLII.

PROBLEM.

TO make a Parallelogram equal to a Triangle given, in an Angle equal to a Right Lined Angle given.

It is required to make a Parallelogram equal to the Triangle ABC, having an Angle equal to the Angle E. Divide the Base BC into two equal parts in D. Draw AG Parallel to BC, (by the 31st.) make also the Angle CDF equal to the Angle E, (by the 23^d.) Then draw the Parallel CG; the figure FDCG is a Parallelogram, seeing the Lines FG, DC, DF, CG, are Parallel. The Angle CDF is equal to the Angle E. I say that the Parallelogram is equal to the Triangle ABC.

Demonstration. The Triangle ABC is half of the Parallelogram FDCG, (by the 41st.) it is also half the Triangle ABC, because the Triangles ADC, ADB, are equal (by the 37th) Therefore the Triangle ABC is equal to the Parallelogram FDCG.

PRO.

PROPOSITION XLIII.
PROBLEM.

The Complements of a Parallelogram are equal.

In the Parallelogram $ABDC$, the complements $AFEH$, $EGDI$, are equal.

Demonstration. The Triangles ABC , BCD are equal (*by the 33d.*) Whence if we Subtract the Triangles HBE , BIE ; FEC , CGE , which are also equal (*by the same.*) the Complements $AFEH$, $GDIE$, which remains, shall be equal.

PROPOSITION XLIV.
PROBLEM.

UPon a Right Line given, to make a Parallelogram at a Right Lined Angle given, equal to a Triangle given.

It is proposed to make a Parallelogram, having one of its Sides equal to the Line D , and one of its Angles equal to the Right Lined Angle E , which must be equal to the Triangle ABC . Make by the preceding Problem the Parallelogram $BFGH$, having the Angle HBF equal to the Angle E , and which may be equal to the Triangle ABC . Continue the Sides

Sides GH , CF , and make HI equal to the Line D ; then draw the Line IBN , and by the Points I and N , draw IL Parallel to GN , and NL parallel to GI ; and continue HB to M , and FB to K . The Parallelogram $KLB M$ is the Parallelogram required.

Demonstration. The Angle $HB F$ equal to the given Angle E , is also equal to the Angle KBM (*by the 15th.*) and the Side $K B$ is equal to the Line HI or D . Lastly, the Parallelogram MK is equal (*by the foregoing*) to the Parallelogram $GFBH$, and this was made equal to the Triangle ABC . Therefore the Parallelogram MK is equal to the Triangle ABC .

U S E.

IN this Proposition is contained a sort Use 44.
of Geometrical Division: For in Arithmetical Division there is proposed a number which may be imagined to be like a Rectangle: For Example, the Rectangle AB containing Twelve square Feet, which is to be divided by another Number, as by two, that is to say, that there must be made another Rectangle equal to the Rectangle AB , which may have BD too, for one of its Sides, and to find how many feet the other Side ought to be, that is to say, the Quotient. One may attain
it

is Geometrically by a Rule and Compass. Take BD of the length of two feet, and draw the Diagonal DEF; the Line AF is that which you seek for. For having made the Rectangle DCFG, the Complements EG, EC, are equal (by the 43d) and EG hath for one of its Sides EH equal to BD two foot, and EI equal to AF. This way of Division is call'd Application, because the Rectangle AB is applyed to the Line BD or EH; and this is the reason why Division is call'd Application; for the Ancient Geometricians did choose rather to make use of a Ruler and Compass, than Arithmetick.

PROPOSITION XLV.

PROBLEM.

UPon a Right-Line to make a Parallelogram equal to a Right-lined Figure given, at a Right-lined Angle given.

There is proposed or given the Right Lined Figure ABCD, and it is required to make a Parallelogram equal thereto; having an Angle equal to the Angle E. Reduce the Right-lined Figure into Triangles, by drawing the Right-line BD; and make (by the 42d.) a Parallelogram FGHI, having the Angle FGH equal to the Angle E, and being in content.

tent equal to the Triangle ABD. Make also (*by the 44th,*) a Parallelogram IHKL, equal to the Triangle BCD, and having a Line equal to IH, and the Angle IHK equal to the Angle E. The Parallelogram FGKL shall be equal to the Right-lined Figure ABCD.

Demonstration. It remains to prove that the Parallelograms FGHI, HKLI, are but one; that is to say that GH, HK make but one streight Line, the Angles GHI, and K, are equal to the Angle E; the Angle K and KHI are equal to Two Right, seeing we have made a Parallelogram KHIL. Whereof the Angles GHI, KHI, are equal to Two Right, and (*by the 14th,*) GH, HK, is one streight Line.

U S E

THis Proposition is in the Practice much like to the former, and serveth to measure the Content of any Figure whatsoever, by reducing it first into Triangles, then making a Right-angled Parallelogram equal thereto. We may also make a Right-angled Parallelogram on a determined or given Line, and which shall be equal to several irregular Figures. Likewise having several Figures, we may make another equal to their difference.

PRO.

PROPOSITION XLVI.
PROBLEM.

Upon a Right Line to describe a Square.

To describe a Square on the Right Line AB, erect two Perpendiculars AC, BD, equal to AB, and draw the Line BD.

Demonstration. The Angles at A and B being Right, the Lines AC, BD, are Parallel (*by the 28th.*) they are also equal: Therefore the Lines AB, CD, are Parallel and equal (*by the 33d.*) and the Angles at A and C, B and D, equal to two Right (*by the 29th.*) and seeing A and B are Right Angles, the Angles C and D shall be also Right. Thence the Figure AD hath all its Sides equal, and all its Angles Right, and consequently is a Square.

PROPOSITION XLVII.
THEOREM.

The Square of the Base of a Right Angled Triangle is equal to the Sum of the Squares of the other two Sides.

It is supposed that the Angle BAC is a Right Angle, and that there be described Squares on the several Sides BC, AB, AC; the Square of the Base BC shall

shall be equal to the Squares of the other two Sides AB , AC . Draw the Line AH Parallel to BD , CE , and draw also the Lines AD , AE , FC , BG . I prove that the Square AF is equal to the Right-angled Figure or long Square BH , and the Square AG , to the Right-angled Figure CH ; and that so the Square BE is equal to the Two Squares AF , AG .

Demonstration. The Triangles FBC , ABD , have their Sides AB , BF ; BD , BC , equal; and the Angles FBC , ABD , are equal: Seeing that each of 'em besides the Right Angle includes the Angle ABC . Thence (*by the 4th*.) the Triangles ABD , FBC , are equal. Now the Square AF is double to the Triangle FBC , (*by the 41st*.) because they have the same Base BF , and are between the same Parallels BF , AC . Likewise the Right-lined Figure BH is double to the Triangle ABD , seeing they have the same Base BD , and are between the same Parallels BD , AH . Therefore the Square AF is equal to the Right-lined Figure BH . After the same manner the Triangles ACE , GCB , are equal (*by the 4th*;) the Square AG is double the Triangle BCG , and the Right-lined Figure CH is double the Triangle ACE ,
(*by*)

(by the 41st.) Thence the Square AG is equal to the Right-lined Figure CH , and by consequence the Sum of the Squares AF , AG , are equal to the Square $BDEC$.

U S E.

Use 47.

IT is said that Pythagoras having found this Proposition, Sacrificed One Hundred Oxen, in thanks to the Muses; it was not without reason, seeing this Proposition serves for a Foundation to a great part of the Mathematicks. For in the first place, Trigonometry cannot be without it, because it is necessary to make the Table, of all the Lines that can be drawn within a Circle, that is to say, of Chords, of Sines, also Tangents and Secants; which I shall here show by one Example.

Let it be supposed that the Semi-Diameter AB be divided into 10000 parts, and that the Arch BC is 30 Degrees. Seeing the Chord or subtendent of 60 Degrees, is equal to the Semi-diameter AC ; BD the Sine of 30 Degrees shall be equal to the half of AC ; it shall therefore be 5000, in the Right-angled Triangle ADB . The Square of AB is equal to the Squares of BD and AD ; make then the Square of AB , by multiplying 10000 by 10000, and from that

that Product Subtract the Square of BD 5000; there remains the Square of AD, or BF, the Sine of the Complement, and extracting the Square, there is found the Line FB. Then, if by the Rule of Three, you say as AD is to BD, so is AC to CE, you shall have the Tangent CE; and adding together the Squares of AC, CE, you shall have (by the 47th.) the Square of AE; and by extracting the Root thereof, you shall have the length of the Line AE the Secant.

Use 47.

We augment figures as much as we please by this Proposition: Example, to double the Square ABCD, continue the Side CD, and make DE equal to AD; the Square of AE shall be the double of the Square of ABCD, seeing that (by the 47th.) it is equal to the Squares of AD and DE. And making a Right Angle AEF, and taking EF equal to AB, the Square of AF shall be Triple to ABCD. And making again the Right Angle AFG, and FG, equal to AB, the Square of AG shall be Quadruple to ABCD. What I here say of a Square, is to be understood of all figures which are alike, that is to say, of the same species.

PRO.

PROPOSITION XLVIII.
THEOREM.

IF the two Squares made upon the Side of a Triangle be equal to the Square made on the other Side, then the Angle comprehended under the two other Sides of the Triangle, is a Right Angle.

If the Square of the Side NP is equal to the Squares of the Sides NL , LP , taken together, the Angle NLP shall be a Right Angle, draw LR Perpendicular to NL , and equal to LP ; then draw the Line NR .

Demonstration. In the Right-angled Triangle NLR , the Square of NR is equal to the Squares of NL , and of LR , or LP , (*by the 47th.*) now the Square of NP is equal to the same Squares of NL , LP ; therefore the Square of NR is equal to that of NP ; and by consequence the Lines NR , NP , are equal. And because the Triangles NLR , NLP , have each of them the Side NL common; and that their Bases RN , NP , are also equal; the Angles NLP , NLR , shall be equal (*by the 8th.*) and the Angle NLR being a Right-angle, the Angle NLP , shall be also a Right-angle.

The End of the First Book.

THE
SECOND BOOK
OF
Euclid's *Elements*:

Euclid Treateth in this Book, of the Power of streight Lines, that is to say, of their Squares, comparing the diuers Rectangles which are made on a Line divided, as well with the Square as with the Rectangle of the whole Line. This part is very useful, seeing it serveth for a Foundation to the Practical Principles of Algebra. The Three first Propositions Demonstrateth the Third Rule of Arithmetick; The Fourth teacheth us to find the Square Root of any number whatsoever; those which follow unto the Eighth, serveth in several accidents happening in Algebra: The remaining Propositions to the end of this Book, are conversant in Trigonometry. This Book

ap.

appeareth at the first sight very difficult, because one doth imagin that it contains mysterious or intricate matters; notwithstanding the greater part of the Demonstrations are founded on a very evident Principle, viz. That the whole is equal to all its parts taken together, therefore one ought not to be discouraged, although one doth not apprehend the Demonstrations of this Book at the first Reading.

D E F I N I T I O N S.

Def 1. of
the Se-
cond
Book.

A Rectangular Parallelogram is comprehended under Two Right Lines, which at their Intersection containeth a Right Angle.

It is to be noted henceforward, that we call that Figure a Rectangular Parallelogram, which hath all its Angles Right; and that the same shall be distinguished as much as is requisite; if we give thereto Length and Breadth, naming only Two of its Lines which comprehendeth any one Angle; as the Lines AB, BC, For the Rectangular Parallelogram ABCD is comprehended under the Lines AB, BC; having BC for its Length, and AB for its Breadth, whence it is not necessary to mention the other Lines, because they are equal to those

those already spoken of. I have already taken notice, that the Line AB being in a Perpendicular Position in respect of BC, produceth the Rectangle ABCD, if moved along the Line BC; and that this Motion representeth Arithmetical Multiplication, in this manner, as the Line AB moves along the Line BC, that is to say, taken as many times as there are Points in BC, Composeth the Rectangle ABCD, wherefore Multiplying AB by BC, I shall have the Rectangle ABCD. As suppose, I know the Number of Mathematical Points there be in the Line AB, for Example, let there be 40, and that in BC there be 60 Points, it is evident that the Rectangle ABCD, shall have as many Lines equal to AB as there are Points in BC, and that by Multiplying 40 by 60, the Product will be 2400, the Number of Mathematical Points contained in the Rectangle ABCD.

I may take what quantity I please for a Mathematical Point, provided I do not subdivide the same; but it is to be noted, that when I measure a Line, I take for a Mathematical Point the Measure which I make use of in measuring the same: For Example, when I say a Line of Five Foot, I take a Line of one

E

Foot

Foot to be a Mathematical Point, without considering that the same is composed of parts. In like manner if I measure a Superficies, the measure I make use of is to me a known Superficie; for example, a square Foot, which I do not subdivide. A Square is rather made use of than any other Figure; because a Square hath its Length equal to its Breadth; whence it is, one is not confined to mention both its dimensions, when I would determine the Area of the Rectangle $ABCD$, I conceive not its sides as if they were single Lines, but as Rectangles of a determined Breadth: For Example, when I say, that the Rectangle $ABCD$ hath the Side AB of Four Foot in Length; seeing that a Foot is as a Mathematical Point, I conceive the Side AB as being one Foot in Breadth; and as the Rectangle $ABEF$. So that by knowing how many times the Breadth BE is contained in BC , I shall know how many times the Line AB is in the Rectangle $ABCD$; that is to say, Multiplying AB , which is Four Foot by BC 6, I shall have 24 Square Foot. Likewise by knowing the superficial content of the Rectangle $ABCD$, which is 24 Square Foot, and one of its Sides AB 4, Dividing the same by 4, giveth the other Side BC 6 Foot. Having

Having drawn the Diameter of a Def. 2. Rectangle, one of the lesser Rectangles, through which the Diameter passeth, together with the Two Complements, is called a Gnomon, as if one should draw the Diameter BD, the Rectangle EG, through which passeth the Diameter, together with the Complements EF, GH, is called a Gnomon because of its Figure, which is that of a Carpenters Square.

PROPOSITION I.

PROBLEM.

IF Two Right Lines be given, and one of them be divided into as many parts or Segments as you please, the Rectangle comprehended under the Two whole Right Lines, shall be equal to all the Rectangles contained under the Line which is not divided, and the several Segments of the divided Line.

Let the Lines proposed be AB, AC, and let AB be divided into as many parts as you please, the Rectangle AD comprehended under the Lines AB, AC, is equal to the Rectangle AG comprehended under AC, AE, and to the Rectangle EH comprehended under EG, equal to AC, and under EF, and to the Rectangle FD, comprehended under FH equal to AC, and under FB.

Demonstration. The Rectangle AD is equal to all its parts taken together, which are the Rectangles AG, EH, and FD. There being no other Rectangle remaining, therefore the Rectangle AD is equal to the Rectangles AG, EH, FD, taken together.

Arithmetically or by Numbers.

THE same Proposition is verified by Numbers. Let us suppose that the Line AC is in Length Five Foot, AE two Foot, EF Four, FB three; and by consequence AB Nine, the Rectangle comprehended under AC Five, and AB Nine, that is to say, Five times Nine or Forty Five, is equal to twice Five or Ten; and Four times Five or Twenty, and Three times Five or Fifteen for Ten, Twenty and Fifteen, added together is equal to Forty Five.

U S E.

A,	53
B,	8
C,	503
B,	8
D,	24
E,	400
F,	424

His Proposition Demonstrates the ordinary practice of Multiplication. For example, let the number A 53, be to be Multiplied by the Line AB, represented by the Number B, 8. I cut or separate the number A, into

as

as many parts as it hath Charallers: For Example, into 50 and 3, which I Multiply by 8, saying 8 times 3 is 24; and by so doing I make a Rectangle. Again, I Multiply the Number 50 by 8; the Product shall be E, 400. It is evident, that the Product of 8 times 53, which is 424, is equal to the Product of 24, and to the Product of 400, they being added or taken together.

PROPOSITION II.
THEOREM.

THE Square of a Line is equal to all the Rectangles comprehended under the whole, and under its parts.

Let AB be the proposed Line, and its Square A B D C. I say, that the Square ABDC is equal to a Rectangle comprehended under the whole Line AB, and under AE; and to a Rectangle comprehended under AB, EF; and to a Third comprehended under AB, and FB.

Demonstration. The Square ABDC is equal to all its parts taken together, which are the Rectangles AG, EH, FD. The first of which is comprehended under AC equal to AB, and under AE. The Second EH, is comprehended un-

der EG equal to AC or AB, and under EF. And the Third FD is comprehended under FH, equal to AB, and under FB: And it being the same thing to be comprehended under a Line equal to AB, as to be comprehended under AB it self. Therefore the Square of AB is equal to all the Rectangles comprehended under AB, and under AE, EF, FB, parts of AB.

ARITHMETICALLY.

LET the Line AB represent the Number Nine; its Square shall be 81. Let the part thereof AE be Four; EF, Three; FB, Two; Nine times Four is Thirty Six, Nine times Three is Twenty Seven, and Nine times Two is Eighteen. It is evident that 36, 27, and 18, added together are 81.

USE.

THIS Proposition serveth to prove Multiplication, as also for Algebraical Equations.

PROPOSITION III.

THEOREM.

IF a Line be divided into Two parts, the Rectangle comprehended under the whole Line and one of the parts, shall be equal to the Rectangle comprehended under the two parts, and the Square of the said part.

the whole Line, and under one of its parts, is equal to the Square of the same part, and to the Rectangle comprehended under both its parts.

Let the Line AB be divided into Two parts in the Point C; and let there be made a Rectangle comprehended under AB, and one of its parts; for example, AC, that is to say, that AD be equal to AC; and compleat of the Rectangle AF, which shall be equal to the Square of AC, and to the Rectangle comprehended under AC, CB. Draw the Perpendicular CE.

Demonstration. The Rectangle AF, comprehended under AB, and under AD, equal to AC, is equal to all its parts, which are the Rectangles AE, CE. The first AE is the Square of AC, seeing that the Lines AC, AD, are equal; and the Rectangle CE is comprehended under CB, and under CE, equal to AD, or AC. Therefore the Rectangle comprehended under AB, AC, is equal to the Square of AC, and to a Rectangle comprehended under AC, CB.

ARITHMETICALLY.

LET AB be 8, AC, 3; CB, 5; the Rectangle comprehended under AB and AC, shall be Three times 8, or 24; the Square of AC, 3, is 9; the Rectangle comprehended under AC, 3; and CB 5, is three times 5, or 15. It is evident, that 15 and 9 are 24.

USE.

A	43
C	40
B	3
120	9
129	

THis Proposition serveth likewise to demonstrate the ordinary practice of Multiplication. For Example, if one would multiply the Number 43 by 3, having separated the Number of 43 into two parts in 40, and 3; three times 43 shall be as much as three times 3, which is Nine, the Square of Three; and Three times Forty, which is 120; for 129 is three times 43. Those which are young beginners ought not to be discouraged, if they do not conceive immediately these Propositions, for they are not difficult, but because they do imagine they contain some great Mystery.

PRO.

PROPOSITION IV

THEOREM

IF a Line be Divided into Two Parts,
the Square of the whole Line shall be
equal to the Two Squares made of its
parts, and to Two Rectangles comprehen-
ded under the same parts.

Let the Line AB be Divided in C,
and let the Square thereof ABDE be
made; let the Diagonal EB be drawn,
and the Perpendicular CF cutting the
same; and through that Point let there
be drawn GL Parallel to AB. It is evi-
dent, that the Square AB DE is equal
to the Four Rectangles GF, CL, CG,
LF. The Two first are the Squares of
AC, and of CB; the Two Complements
are comprehended under AC, CB.

Demonstration. The Sides AE, AB,
are equal; thence the Angles AEB, ABE,
are half Right; and because of the Pa-
rallels GL, AB, the Angles of the Tri-
angles of the Square GE, (*by the 29th*)
shall be equal; as also the Sides (*by
the 6th of the 1st*) Thence GF is the
Square of AC. In like manner the Re-
ctangle CL is the Square of CB; the
Rectangle GC is comprehended under
AC, and AG equal to BL, or BC; the
Rectangle LF is comprehended under

E 5

LD

The Second Book of

LD equal to AC, and under FD, equal to BC.

Coroll. If a Diagonal be drawn in a Square, the Rectangles through which it passeth are Squares.

U S E.

A,	144
B,	22
C,	12

THis Proposition giveth us the practical way of finding or extracting the Square Root of a Number propounded: Let the same be the number A, 144, represented by the Square AD, and its Root by the Line AB. Moreover I know that the Line required AB must have Two Figures: I therefore imagine that the Line AB is Divided in C, and that AC representeth the first Figure, and BC the Second. I seek the Root of the First Figure of the Number 144, which is 100, and I find that it is 10, and making its Square 100, represented by the Square GF, I Subtract the same from 144, and there remains 44 for the Rectangle GC, FL, and the Square CL. But because this Gnomonical Figure is not proper, I transport the Rectangle FL, in KG, and so I have the Rectangle KL containing 44. I know also almost all the Length of the Side KB, for AC is 20, therefore KC is 20: I must then
Di

Divide 144 by 20 as abides as we say, and
find the Divisor, I double the Rem. square
and I say being many times 20 in 244 it
find it 2 times, for the Side BL, but
because 20 was not the whole Side KB
but only KC, this 2 which comes in is the
Quotient in 20 be added in the Divisor,
which they will know. So I find the same
2 times precisely in 44, the Square Root
there shall be 12. You see that the Square
of 144 is equal to the Square of 10, 10
the Square of 2, which is 4, and so twice
20, which are Two Rectangles compre-
hended under 20 and under 10.

PROPOSITION V.

THEOREM.

IF A Right Line be cut into equal parts,
and into unequal parts, the Rectan-
gles comprehended under the unequal
parts, together with the Square which
is of the middle part, the difference betwixt
the parts, is equal to the Square of half
the Line.

If the Line AB is Divided equally
in C, and unequally in D; the Rect-
angle AH comprehended under the Seg-
ments AD, DB, together with the
Square of CD, shall be equal to the
Square of CF, that is of half of AB, viz.
GB. Make an end of the Figure as you

see.

see it; the Rectangles LG, DI, shall be Squares (by the Coroll. of the 4th). I prove that the Rectangle AH, comprehended under AD, and DH, equal to DB, with the Square LG, is equal to the Square CF.

Demonstration. The Rectangle AL, is equal to the Rectangle DF, the one and the other being comprehended under half the Line AB, and under BD, or DH equal thereto. Add to both the Rectangle CH; the Rectangle AH, shall be equal to the Gnomon LBG. Again: to both add the Square LG; the Rectangle AH, with the Square LG, shall be equal to the Square CF.

ARITHMETICALLY.

L Et AB be 10, AC is 5, as also CB. Let CD be 2, and DB, 3; the Rectangle comprehended under AD, 7, and DB, 3; that is to say 21, with the Square of CD 2, which is 4, shall be equal to the Square of CB 5, which is 25.

USE.

T His Proposition is very useful in the Third Book; we make use thereof in Algebra, to Demonstrate the way of finding the Root of an affected Square or Equation.

PRO.

PROPOSITION VI.
THEOREM.

IF one add a Line to another which is Divided into Two equal parts; the Rectangle comprehended under the Line compounded of both, and under the Line added, together with the Square of half the divided Line, is equal to the Square of a Line compounded of half the Divided Line, and the Line added.

If one add the Line BD, to the Line AB, which is equally Divided in C; the Rectangle AN, comprehended under AD, and under DN, or DB, with the Square of CB, is equal to the Square of CD. Make the Square of CD, and having drawn the Diagonal FD, draw BG Parallel to FC, which cuts FD in the Point H, through which passeth HN Parallel to AB: KG shall be the Square of BC; and BN, that of BD.

Demonstration. The Rectangles AK, CH, on equal Bases, AC, BC, are equal (by the 38th of the 1st.) The Complements CH, HE are equal (by the 43d of the 1st.) Therefore the Rectangles AK, HE, are equal. Add to both the Rectangle CN, and the Square KG, the Rectangles AK, CN, that is to say the Rectangle AN with the Square KG, shall

shall be equal to the Rectangles CN, HE, and to the Square KG, that is to say to the Square CE.

Arithmetically or by Numbers.

Et AB be 8, AC 4, CB 4, BD 3, then AD shall be 7. It is evident that the Rectangle AN, three times 11, that is to say 33, with the Square of KG 16, which together are 49, is equal to the Square of CD 7, which is 49, for 7 times 7 is 49.

USE

Fig. 6.

Manrolytus measured the whole Earth by one single Observation, making use of this Proposition. He would that one should observe from A the top of a Mountain whose height is known, the Angle BAC, which is made by the Line AB which toucheth the Superficies of the Earth, and the Line AC which passeth through the Center, and that in the Triangle ADE, knowing the Angle A, and the Right Angle ADE, one findeth by Trigonometry the Sides AE, ED, and because that it is easie to Demonstrate that ED, EB, are equal, one shall know the Length of the Line AB, and its Square. Now having Demonstrated by the preceding Proposition, that the

the Line ED being Divided into two equal parts in the Point C and having added thereto AD , the Rectangle comprehended under EA , and under AD , together with the Square of CD , or CB , is equal to the Square of AC , and the Angle ABC being Right, (as it is proved in the Third Book) the Square of AC is equal to the Square of AB , BC . Therefore the Rectangles EA , AD together with the Square of BC is equal to the Squares of AB , BC . Take from both the Square of BC , the Rectangle under AE , AD , shall be equal to the Square of AB . Divide then the Square of AB , which you know by the Height of the Mountain, which is AD , the Quotient shall be the Line AE , from which must be subtracted the Height of the same Mountain, and you shall have DE , or the Diameter of the Earth. or $Imposz$ AD but AD or

We make use of the same Proposition in Algebra; as also to Demonstrate the practice of finding the Root of a Square equal to a Number increased more by a certain number of Roots. The Two Propositions following serve also to prove other like Practices.

PRO

PROPOSITION VII.

THEOREM.

IF one Divides a Line, the Square of the whole Line, together with that of one of its parts, shall be equal to Two Rectangles comprehended under the whole Line, and that first part, and to the Square of the other part.

Let the Line AB be divided at discretion, in the Point C; the Square AD of the Line AB, together with the Square AL, shall be equal to Two Rectangles comprehended under AB, AC. With the Square of CB make the Square AB; and having drawn the Diagonal EB, and the Lines CF, HGL, prolong EA, and make AK equal to AC; so shall AL be the Square of AC, and HK shall be equal to AB. For HA is equal to GC; and GC is equal to CB, seeing CI is the Square of CB, (by the Coroll. of the 4th.)

Demonstration. It is evident that the Squares AD, AL, are equal to the Rectangles HL, HD, and to the Square of CL. For the Rectangle HL is comprehended under HK equal to AB, and under KL equal to AC. In like manner, the Rectangle HD is comprehended under HL, equal to AB, and under HE equal

equal to AC. Therefore the Squares of AB, AC, are equal to Two Rectangles comprehended under AB, AC, and to the Square of CB.

ARITHMETICALLY.

Let it be supposed that the same AB contains 9 equal parts; AC, 4; CB, 5: the Square of AB, 9, is 81; the Square of AC, 4, is 16. Which being added together is 97. A Rectangle under AB, AC, or 4 times 9 is 36; which doubled is 72; The Square of CB, 5, is 25. Now 25 and 72 is also 97.

PROPOSITION VIII.
THEOREM.

IF one Divide a Line, and that thereto one add one of its parts, the Square of the Line Composed shall be equal to Four Rectangles, comprehended under the First Line, and under the part added together with the Square of the other part.

Let the Line AB be Divided at discretion in the Point C; and let thereto be added BD, equal to CB, the Square of AD shall be equal to 4 Rectangles comprehended under AB, BC or BD, and to the Square of AC. Let the Square
of

of AD be made, and having drawn the Diagonal AE, draw the Perpendiculars BP, CN, which cut the Diagonal in L and O; draw also the Lines MH, GLR, Parallel to AB; the Rectangles GC, LK, PH, MB, NR, shall be Squares (by the Coroll. of the 4th.)

Demonstration. The Square ADEF is equal to all its parts, the Rectangles IB, OD, PM, are comprehended under Lines equal to AB and CB. If you add the Rectangle ML to the Rectangle PH; you shall have a Rectangle comprehend- ed under one Line equal to AB, and under another equal to CB or BD. There remains only the Square GC, which is that of AC. Therefore the Square of AD is equal to Four Rectan- gles comprehended under AB, BD, and to the Square of AC.

ARITHMETICALLY.

Let the Line AB contain 7 parts, AC 3, CB 4, as well as BD, the Square of AD, 11, shall be 121. A Re- ctangle under AB, 7; and BD, 4; is Twenty Eight: which takes 4 times is 112, and the Square of 3 is 9. Now 112, and 9, added, are 121. **PRO.**

PROPOSITION IX.

THEOREM.

IF a Line be equally Divided, and unequally, the Squares of the unequal parts shall be double to the Squares of half the Line, and of the part between.

Let the Line AB be Divided equally, in the Point C, and unequally, in the Point D. The Squares of the unequal parts AD, DB, shall be double of the Squares of AC, which is the half of AB, and the Square between both CD. Draw AB, and the Perpendicular CF, equal to AC: Draw also the Lines AE, BE, and the Perpendicular DF, as also FG Parallel to CD. Then draw the Line AF.

Demonstration. The Lines AC, CE, are equal, and the Angle C is Right; thence (by the 6th of the 1st) the Angles CAE, CEA, are equal to a half Right Angle. In like manner, the Angles CEB, CBE, GFE, DEB, are half Right; the Lines GF, GE, DF, DB, are equal, and the total Angle AEF is Right. The Square of AE (by the 47th of the 1st) is equal to the Squares of AC, CE, which are equal: Thence it is double to the Square of AC. After the same manner the Square of EF is double to

the Square of GF, or CD: Now the Square of AF is equal to the Squares of AE, EF; seeing that the Angle AEF is Right: Therefore the Square of AF is double to the Squares of AC, CD. The same Square AF is equal to the Squares of AD, DE, or DB; seeing that the Angle D is Right: Therefore the Squares of AD, DB, are double to the Squares of AC, CD.

ARITHMETICALLY.

L Et AB be 10; AC, 5; CD, 3; DB, 2: The Squares of AD, 8, and DB, 29, that is to say 64, and 4, which together are 68, are double to the Square AC, 5, which is 25; and of the Square CD, 3, which is 9; for 25 and 9 are 34, which doubled make 68.

U S E.

I Have not found this Proposition, nor the ensuing, but only in Algebra.

PROPOSITION X.

THEOREM.

IF one add a Line, to another which is equally Divided; the Square of the Line Composed of both, with the Square of the Line added, are double to the Square of half the Line, and to the Square which is Composed of the half Line and the Line added. If

If one supposeth AB , to be Divided in the middle at the Point C , and if thereto be added the Line BD , the Squares of AB and BD , shall be double to the Squares of AC , and CD , added together. Draw the Perpendiculars CE , DF , equal to AC : Then draw the Lines AE , EF , AG , EBG .

Demonstration. The Lines AC , CE , CB , being equal, and the Angles at the Point C being Right: The Angles AEC , CEB , CBE , DBG , DGB , shall be half Right; and the Lines DB , DG , and EF , FG , CD , shall be equal. The Square of AE , is double to the Square of AC ; the Square of EG , is double to the Square of EF , or CD , (*by the 47th of the 1st*) Now the Square of AG , is equal to the Squares of AE , EG , (*by the 47th of the 1st*.) Therefore the Square of AG is double to the Squares of AC , CD . The same AG (*by the 47th of the 1st*.) is equal to the Squares of AD , BD , or GD : Therefore the Squares of AD , BD , are double the Squares of AC , CD .

ARITHMETICALLY.

L Et AB be 6 parts, AC , 3; CB , 3;
 BD , 4: the Square of AD , 10,
 is 100, the Square of BD , 4, is 16,
 which

The Second Book of

which are 116. The Square of AC, 3, is 9; the Square of CD, 7, is 49. Now 49, and 9, is 58, the half of 116.

PROPOSITION XI

PROBLEM.

TO Divide a Line, so that the Rectangle comprehended under the whole Line, and under one of its parts, shall be equal to the Square of the other part.

It is proposed to Divide the Line AB, so that the Rectangle comprehended under the whole Line AB, and under HB, be equal to the Square of AH. Make a Square of AB (by the 46th of the 1st;) Divide AD in the middle in E; then Draw EB, and make EF equal to EB. Make the Square AF, that is to say, that AF, AH, be equal. I say that the Square of AH, shall be equal to the Rectangle HC, comprehended under HB, and the Line BC, equal to AB.

Demonstration. The Line AD is equally divided in E, and there is added thereto the Line FA; thence (by the 6th) the Rectangle DG, comprehended under DE and FG, equal to AF, with the Square of AE, is equal to the Square of EF, equal to EB. Now the Square of EB is equal to the Square of AB, AE,

AE; (by the 47th of the 1st) therefore the Squares of AB, AE, are equal to the Rectangle DG, and to the Square of AE: and taking away from both the Square of AE, the Square of AB, which is AC, shall be equal to the Rectangle DG: taking also away the Rectangle DH, which is in both, the Rectangle HC shall be equal to the Square of AG.

USE.

THis Proposition serveth to cut a Line in extreame and mean Proportion; as shall be shewn in the Sixth Book. It is used often in the 14th of Euclid's Elements, to find the Sides of Regular Bodies. It serveth for the 10th of the Fourth Book, to inscribe a Pentagon in a Circle, as also a Pentadecagone. You shall see other uses of a Line thus divided, in (the 30th of the Sixth Book.)

PROPOSITION XII.

THEOREM.

IN an obtuse angled Triangle the Square of the side opposite to the obtuse Angle, is equal to the Squares of the other two sides, and to two Rectangles comprehended under the side on which one draweth a Perpendicular, and under the Line which

which is between the Triangle, and that Perpendicular.

Let the Angle ACB of the Triangle ABC, be obtuse; and let AD be drawn perpendicular to BC. The Square of the side AB, is equal to the Squares of the sides AC, CB, and to two Rectangles comprehended under the side BC, and under DC.

Demonstration. The Square of AB is equal to the Squares of AD, DB. (*by the 47th of the 1st*) the Square of DB is equal to the Squares of DC and CB, and to two Rectangles comprehended under DC, CB, (*by the 4th.*) therefore the Square of AB is equal to the Squares of AD, DC, CB, and to two Rectangles comprehended under DC, CB, in the place of the two last Squares AD, DC. Put the Square of AC, which is equal to them (*by the 47th of the 1st*) the Square of AB shall be equal to the Square of AC and CB, and to two Rectangles comprehended under DC, CB.

U S E.

THis Proposition is useful to measure the Area of a Triangle, its three sides being known: for Example; If the Side AB was twenty Foot; AC, 13; BC, 11; the Square of AB would be four hun-

hundred; the Square of AC, one hundred sixty nine; and the Square of BC, one hundred twenty one: the Sum of the two last is, Two hundred and ninety; which being subtratted from four hundred, leaves one hundred and ten for the two Rectangles under BC, CD, the one half fifty five shall be one of those Rectangles: which divided by BC 11, we shall have five for the Line CD, whose Square is twenty five; which being subtratted from the Square of AC, one hundred sixty nine; there remains the Square of AD one hundred forty four: and its Root shall be the side AD: which being multiplied by $5\frac{1}{2}$, the half of BC, you have the Area of the Triangle ABC, containing 66 square Feet.

PROPOSITION XIII.

THEOREM.

IN any Triangle whatever, the Square of the side opposite to an acute Angle, together with two Rectangles comprehended under the side on which the Perpendicular falleth, and under the Line which is betwixt the Perpendicular and that Angle, is equal to the Square of the other sides.

Let the proposed Triangle be ABC, which hath the Angle C, acute; and if one draw AD perpendicular to BC, the Square of the side AB, which is opposite

F

site

site to the acute Angle C, together with two Rectangles comprehended under BC, DC, shall be equal to the Squares of AC, BC.

Demonstr. The Line BC is divided in D; whence (by the 7th) the Square of BC, DC, are equal to two Rectangles under BC, DC, and to the Square of BD: add to both the Square of AD; the Square of BD, DC, AD, shall be equal to two Rectangles under BC, DC; and to the Squares of BD, AD, in the place of the Squares of CD, AD, put the Square of AC, which is equal to them (by the 47th of the 1st) and instead of the Squares of AD, AB, substitute the Square of AB, which is equal to them, the Square of BC, AC, shall be equal to the Square of AB, and to two Rectangles comprehended under BC, DC.

U S E.

These Propositions are very necessary in Trigonometry: I make use thereof in the eighth Proposition of the third Book, to prove, That in a Triangle there is the same Reason between the whole Sine and the Sine of the Angle, as are between the Rectangle of the sides comprehending that Angle, and the double Area of the Triangle. I make use thereof in several other Propositions, as in the seventh.

PRO-

PROPOSITION XIV.
PROBLEM.

TO describe a Square equal to a right lined Figure given.

To make a Square equal to a Right lined Figure A, make (*by the 45th of the 1st*) a Rectangle BC, DE, equal to the Right lined Figure A: if its sides CD, DC, were equal, we should have already our desire; if they be unequal, continue the Line BC, until CF be equal to CD; and dividing the Line BF in the middle, in the Point G describe the Semicircle FHB, then continue DC to H, the Square of the Line CH is equal to the Right lined Figure A: draw the Line GH.

Demonstration. The Line BF is equally divided in G, and unequally in C; thence (*by the 5th*) the Rectangle comprehended under BC, CF, or CD, that is to say, the Rectangle BD, with the Square CG, is equal to the Square of GB, or to its Equal GH. Now (*by the 47th of the 1st*) the Square of GH is equal to the Square of CH, CG, therefore the Rectangle BD, and the Square of CG, is equal to the Squares of CG, and of CH; and taking away the Square of CG, which is common to both, the

Rectangle BD, or the Right lined Figure A, is equal to the Square of CH.

U S E.

THis Proposition serveth in the first place, to reduce into a Square any Right lined Figure whatever: and whereas a Square is the first Measure of all Superficies, because its Length and Breadth is equal, we measure by this means all right lined Figures. In the second place this Proposition teacheth us to find a mean Proportion between two given Lines; as we shall see in that Thirteenth Proposition of the Sixth Book.

This Proposition may also serve to square curve lined Figures, and even Circles themselves; for any crooked or curve lined Figure may, to sence, be reduced to a Right lined Figure; as if we inscribe in a Circle a Polygon having a thousand sides, it shall not be sensibly different from a Circle: and reducing the Polygon into a Square, we square nearly the Circle.

The end of the Second Book.

THE

THE
THIRD BOOK
OF

Euclid's *Elements* :

THis third Book explaineth
the Propriety of a Circle;
and compareth the divers
Lines which may be drawn
within and without its Circumference.
It farther considereth the Circumstances
of Circles, which cut each other, or which
touch a streight Line; and the different
Angles which are made, as well those
in their Centers as in their Circumfe-
rences. In fine, it giveth the first Prin-
ciples for establishing the Practice of
Geometry; by the which we make use,
and that very commodiously, of a Circle
in almost all Treatises in the Mathe-
maticks.

DEFINITIONS.

Def. of the
3 Book.

1. Those Circles are equal, whose Diameters, or Semidiameters, are equal.

Fig. 1.

2. A Line toucheth a Circle when meeting with the Circumference thereof it cutteth not the same; as the Line A B.

Fig. 2.

3. Circles touch each other, when meeting they cut not each other, as the Circles A B and C.

Fig. 3.

4. Right Lines in a Circle are equally distant from the Center, when Perpendiculars drawn from the Center to those Lines are equal: As if the Lines E F, E G, being Perpendiculars to the Lines A B, C D, are equal, A B, C D, shall be equally distant from the Center; because the distance ought always to be taken (or measured) by Perpendicular Lines.

Fig. 4.

5. A Segment of a Circle is a Figure terminated on the one side by a straight Line, and on the other by the Circumference of a Circle; as L O N, L M N.

6. The Angle of a Segment is an Angle which the Circumference maketh with a straight Line; as the Angle O L N, L M N.

7. An

7. An Angle is said to be in a Segment of a Circle, when the Lines which from the same are therein; as the Angle FGH , is in the Segment FGH .

8. An Angle is upon that Arch to which it is opposite, or to which it serveth for a Base; as the Angle FGH , is upon the Arch FIH , which may be said to be its Base.

9. A Sector is a Figure comprehended under two Semidiameters, and under the Arch which serveth to them for a Base; as the Figure $FIGH$.

PROPOSITION I.

PROBLEM.

To find the Center of a Circle.

IF you would find the Center of the Circle $AEBD$, draw the Line AB , and divide the same in the middle, in the Point C ; at which Point erect a Perpendicular ED , which you shall divide also equally in the Point F . This Point F shall be the Center of the Circle; for if it be not, imagine, if you please, that the Point G is the Center: Draw the Lines GA , GB , GC .

Demonstration. If the Point G were the Center, the Triangle GAC , GBC , would have the sides GA , GB , equal by the definition of a Circle: AC , CB , are equal to the Line AB , having been divided in the middle, in the Point C . And CG being common; the Angles GCB , GCA , would then be equal (by the 8th of the 1st) and CG would be then a Perpendicular, and not CD ; which would be contrary to the Hypothesis. Therefore the Center cannot be out of the Line CD . I further add; that it must be in the Point F , which divideth the same into two equal Parts; otherwise the Lines drawn from the Center to the Circumference would not be equal.

Corollary. The Center of a Circle is in a Line which divideth another Line in the middle, and that perpendicularly.

U S E.

This first Proposition is necessary to demonstrate those which follow.

PRO

PROPOSITION III

THEOREM

A *Straight Line drawn from one point of the Circumference of a Circle to another, shall fall within the same.*

Let there be drawn from the Point B, in the Circumference, a Line to the Point C. I say, that it shall fall wholly within the Circle. To prove that it cannot fall without the Circle, as BVC; having found the Center thereof, which is A, draw the Lines AB, AC, AV.

Demonstration. The Sides AB, AC, of the Triangle ABC, are equal: whence (by the 5th of the 18th.) the Angles ABC, ACB, are equal. And, seeing the Angle AVC, is exterior in respect of the Triangle AVB, it is greater than ABC, (by the 16th of the 1st.) it shall be also greater than the Angle ACB. Thence (by the 19th of the 1st) in the Triangle ACV, the side AC, opposite to the greatest Angle AVC, is greater than AV; and by consequence AV cannot reach the Circumference of the Circle, seeing it is shorter than AC, which doth but reach the same; where-

fore the Point V is within the Circle : the same may be proved of any Point in the Line AB ; and therefore the whole Line AB falls within the Circle.

U S E.

IT is on this Proposition that are grounded those which demonstrate that a Circle toucheth a straight Line but only in one Point: for if the Line should touch two Points of its Circumference, it would be then drawn from one Point of its Circumference to the other ; and consequently, would fall within the Circle, according to this Proposition ; although its Definition saith, that it cutteth not its Circumference. Theodosius demonstrateth after the same manner, That a Globe cannot touch a Plain but in one single Point ; otherwise the Plain would enter into the Globe.

PROPOSITION III. THEOREM

IF the Diameter of a Circle cutteth a Line which passeth not through the Center, into two equal parts, it shall cut the same at right Angles ; and if it cutteth it at Right Angles, it divideth it into two equal Parts.

If the Diameter AC cut the Line BD, which passeth not through the Center F, into two equal parts in E, it shall cut the same at Right Angles. Draw the Lines FB, FD.

Demonstration. In the Triangles FEB, FED, the side FE is common; the sides BE, ED, are equal, seeing the Line BD is divided equally in the Point E; the Bases FB, FD, are equal: thence (by the 8th of the 1st.) the Angles BEF, DEF, are equal, and consequently right. Moreover, I say, that if the Angles BEF, DEF, are right, the Line BD shall be divided into two equal parts in E; that is to say, the Lines BE, ED, shall be equal.

Demonstration. The Triangles BEF, DEF, are Rectangular: thence (by the 47th of the 1st.) the Square of the Side FD shall be equal to the Squares of ED, and of EF: in like manner, the the Square of BF shall be equal to the Squares of BE, EF. Now the Squares of BF, FD, are equal, because the Lines are equal; therefore the Squares of BF, EF, are equal to the Squares of ED, EF; and taking away EF, the Squares of BE, ED, shall be equal, and consequently the Lines.

PRO.

PROPOSITION IV.

THEOREM.

Two Lines drawn in a Circle, cut not each other equally, if the Point of their Interfection be not in the Center.

If the Lines AC, BD, cut each other in the Point I, which is not the Center of the Circle, they will not cut each other equally. In the first place, if one of them should pass through the Center, it is evident, that the same cannot be divided into two equal parts, but by the Center. If neither of them pass through the Center, as BD, EI, draw the Line AIC, which passeth through the Center.

Demonstration. If the Line AC divide the Line BD into two equal Parts in I, the Angles AID, AIB, would be equal (by the 2d.) In like manner, if the Line EG was divided into two equal Parts, in the Point I, the Angle AIG would be right: thence the Angles AIB, AIG, would be right, and consequently equal, which cannot be, the one being part of the other. Lastly, the Line AIC, which passeth through the Centers, would then be perpendicular to the Lines BD, GI, if both of them were divided equally in the Point I.

USE.

PROPOSITION VI.
THEOREM.

BOTH of those Propositions are of Use in Trigonometry: whereby is demonstrated, that the half Chord of an Arch is perpendicular to the Semidiameter; and by consequence, it is the sine of the half Arch: by which also it is demonstrable, that the Side of a Triangle hath the same Proportion as the sines of their opposite Angles. We furthermore make use of this Proposition to find the Eccentricity of the Circle which the Sun describeth in a Year.

PROPOSITION V.
PROBLEM.

Circles which cut each other have not the same Center.

The Circles ABC , ADC , which cut each other in the Points A and C , have not the same Center: for if they had the same Center E , the Lines EA , ED , would then be equal (by the definition of a Circle) as also the Lines EA , EB : thence the Lines ED , EB , would be equal; which is impossible, the one being a part of the other.

PRO-

PROPOSITION VI. THEOREM.

IF two Circles inwardly touch one the other, they have not one and the same Center.

The Circles BD, BC, which touch inwardly one the other, in the Point B, have not the same Center: for if the Point A was the Center of both Circles, the Lines AB, AD, and AB, AC, would be equal (by the Definition of a Circle) and so AD, AC, would be equal; which is impossible, the one being a part of the other.

PROPOSITION VII. THEOREM.

IF in a Circle several Lines be drawn from a Point which is not the Center, unto the Circumference; first, that which passeth through the Center shall be greatest; secondly, the least shall be the Remainder of the same Line; thirdly, the nearest Line to the greatest is greater than any Line which is farther from it; fourthly, there cannot be drawn more than two equal Lines.

Let there be drawn several Lines from the Point A, which is not the Center of the Circle, to the Circumference;

rence; and let the Line AC pass through the Center B : I demonstrate that it shall be the greatest or longest, that it is greater than AF . Draw EB .

Demonstration. The sides AB , BF , of the Triangle ABF , are greater than the side AF , (by the 20th of the 1st.) Now BF , BC , are equal (by the definition of a Circle) therefore AB , BC , that is to say, AC , is greater than AF . Moreover, in the second place, I say, that AD is the shortest; for Example, shorter than AE . Draw EB .

Demonstration. The sides EA , AB , of the Triangle ABE , are greater than BE , which is equal to BD : thence EA , AB , are greater than BD ; and taking away AB , which is common, AE shall be greater than AD . Moreover, AF , which is nearer to AC than AE , is greater than AE .

Demonstration. The Triangles FBA , EBA , have the Sides BF , BE , equal, and BA common to both: the Angle ABF is greater than the Angle ABE ; thence (by the 24th of the first) AF is greater than AE .

In fine, I say that there cannot be drawn but two equal Lines from the Point A , to the Circumference, let the Angles ABE , ABG , be equal, and let be drawn the Line AE , AG . De-

Demonstration. The Triangles ABG , ABE , have their Sides AE , AG , equal; and all the Lines which may be drawn on either side of these, shall be either nearer to AC than the Lines AE , AG , or farther: and so they shall be either shorter or longer than AE , AG . Therefore, there cannot be drawn more than two equal Lines.

U. S. E.

Theodofius doth very well make use of this Proposition, to prove, that if from a Point in the Superficies of a Sphere, which is not the Pole of a Circle, there be drawn several Arks of great Circles unto the Circumference of a Circle: that which passeth through the Pole of that Circle to which the great Circles are drawn, is greatest. For Example, if from the Pole of the World, which is not the Pole of the Horizon (for the Zenith is the Pole thereof) there be drawn several Arks of great Circles unto the Circumference of the Horizon: the greatest Ark of all shall be that part of the Meridian which passeth through the Zenith. This Proposition is also made use of, to prove, that the Sun being in Apoge, is farthest distant from the Earth.

PROPOSITION VIII.
THEOREM.

IF from a Point without a Circle, be drawn several Lines to its Circumference; first, of all those which are drawn to its concave Circumference, the greatest passeth through the Center; secondly, those nearest thereto are greater than those which are farther off; thirdly, amongst the Lines which fall on the convex Circumference, the least being continued, passeth through the Center; fourthly, the nearest thereto are least; fifthly, there cannot be drawn but only two equal, whether they be drawn to the convex Circumference, or that they fall on the concave.

Let there be drawn from the Point A, several Lines to the Circumference of the Circle GC, DE. In the first place the Line AC, which passeth through the Center B, is the greatest of all those which fall on the concave Circumference: For Example, it is greater than AD. Draw the Line BD.

Demonstration. In the Triangle ABD the sides AB, BD, are greater than the Side AD; now the Sides AB, BC, are equal to AB, BD; thence AB, BC, or AC, is greater than AD.

2. AD

2. AD is greater than AE.

Demonstration. The Triangles ABD, ABE, have the Side AB common, and the Sides BE, BD, equal; and the Angle ABD, is greater than the Angle ABE: thence (by the 24th of the 1st.) the Base AD is greater than the Base AE.

3. AF, which being prolonged passeth through the Center, is the least of those Lines which are drawn to the convex Circumference LFIK. For example, it is less than AI. Draw IB.

Demonstration. The sides AI, IB, are greater than AB, (by the 20th of the 1st.) wherefore taking away the equal Lines BI, BF, AF, shall be less than AI.

4. AI is less than AK. Draw the Line BK.

Demonstration. In the Triangles AIB, AKB, the sides AK, KB, are greater than the Sides AI, IB, (by the 21st of the 1st.) wherefore taking away the equal sides, BK, BI, there remains AI less than AK.

5. There can be drawn but two equal Lines, make the Angles ABL, ABK, equal.

Demonstration. The Triangles ABL, ABK, shall have their Bases AL, AK, equal (by the 4th of the 1st.) but there cannot

cannot be drawn any other, which will not be either nearer or farther from AF, and which will not be either greater or lesser than AK, AL.

PROPOSITION IX. THEOREM.

The Point from whence may be drawn to the Circumference of a Circle, three equal Lines, as its Center.

If that Point were not the Center of the Circle, there could be drawn therefrom only two equal Lines (by the 7th.)

PROPOSITION X. THEOREM.

Two Circles cut each other only in two Points.

If the two Circles ABD, ABFD, did cut each other in three Points, A, B, D; seek (by the 2d) the Center C of the Circles AE, BD, and draw the Lines CA, CB, CD.

Demonstration. The Lines AC, BC, DC, drawn from the Center C, to the Circumference of the Circle AE, BD, are equal: Now the same Lines are also drawn to the Circumference of the Circle AB, FB: thence (by the 9th) the Point C shall be the Center of the Circle ABFD

ABFD. So two Circles which cut each other shall have the same Center; which is contrary to the Fifth Proposition.

PROPOSITION XI. THEOREM.

IF Two Circles inwardly touch one the other, the Line drawn through both Centers, passeth also through the Point, where they touch one the other.

If the two Circles EAB, EFG, inwardly touch one the other, in the Point E, the Line drawn through both Centers shall pass through the Point E. Now if the Point D be the Center of the lesser Circle, and C that of the greater, in such manner, that the Line CD passeth not through the Point E, draw the Lines CE, DE.

Demonstration. The Lines DE, DG, drawn from the Center D of the lesser Circle, are equal; and adding the Line DC; the Lines ED, DC, should be equal to CG. Now ED, DC, are greater than EC. (by the 20th of the 1st) so that CG is greater than CE, and notwithstanding C being the Center of the great Circle, CE, CB, are equal; thence CG would be greater than CB, which is impossible.

Q.E.D.

PRO-

PROPOSITION XII.

THEOREM.

IF two Circles touch one the other outwardly, the Line drawn through their Centers pass through the Point where they touch one the other.

If the Line AB drawn from the Center A , to the Center B , passeth not through the Point C , where the Circles touch one the other; draw the Lines AC , BC .

Demonstration. In the Triangle ACB , the Side AB would be greater than AC , BC ; for BC , BE , are equal, as well as AD , AC ; which would be contrary to the 20th Proposition of the 1st.

PROPOSITION XIII.

THEOREM.

TWO Circles touch one the other only in one Point.

In the first place, if two Circles touch one the other inwardly, they shall touch one the other only in one Point C , in the Line BAC , which passeth through their Centers A and B . Now if it be supposed that they touch one the other again in the Point D ; draw the Lines AD , DB .

Demonstration. The Lines AD , AC ,
being

being drawn from the Center of the lesser Circle, are equal, and adding to both AB the Lines BA, AC, BA, AD should be equal: Now BC, BD being drawn from the Center of the greater Circle, should be also equal; thence the Sides BA, AD, should be equal to the Side BD: which is contrary to the 20th of the 1st.

Secondly, if the Two Circles touch outwardly; drawing the Line AB from one Center to the other, it shall pass through the Point C, where the Circles touch (*by the 12th.*) Now if you say they also touch in the Point D; having drawn the Lines AD, BD; the Lines BD, BC; AD, AC, being equal, the Two Sides of a Triangle taken together should then be equal to the Third, which is contrary to the 20th of the 1st.

U S E.

These Four Propositions are very clear, they are notwithstanding necessary in Astronomy, when we make use of Epicycles to explain the motion of the Planets.

PROPOSITION XIV.

THEOREM.

Equal Right Lines drawn in a Circle, are equally distant from the Center, and

and Right Lines, which are equally distant from the Center, are equal.

If the Lines AB , CD , are equal; I demonstrate that the Perpendicular EF , EG , drawn from the Center, are also equal. Draw the Lines EA , EC .

Demonstration. The Perpendiculars EF , EG , divideth the Lines AB , CD , in the middle in F and G (by the 3d :) So AF , CG , are equal, the Angles F and G are Right; thence (by the 47th of the 1st) the Square of EA is equal to the Squares of EF , FA ; as the Square of EC is equal to the Squares of CG , EG : Now the Squares of EA , EC , are equal, the Lines EA , EC , being equal: Therefore the Squares of EF , FA , are equal to the Squares of EG , GC ; and taking away the equal Squares AF , CG ; there remains the equal Squares EG , EF ; so then the Lines EG , EF , which are the distances, are equal.

If you suppose that the distances, or perpendiculars EG , EF , are equal; I would demonstrate after the same manner that the Squares of EF , FA , are equal to the Squares of EG , GC , and taking away the equal Squares of EF , EG , there shall remain the equal Squares AF , CG ; so then the Lines AF , CG , and their double AB , CD , are equal.

PRO.

PROPOSITION XV.
THEOREM.

The Diameter is the greatest Line drawn in a Circle; and those nearest the Center are greatest.

The Diameter AB is the greatest of the Lines that are drawn in the Circle; for example, it is greater than CD. Draw the Lines EC, ED.

Demonstration. In the Triangle CED, the Two Sides EC, ED, are greater than the Side CD (by the 20th of the 1st.) Now AE, EB, or AB is equal to EC, ED: Thence the Diameter AB is greater than CD.

Secondly, Let the Line GI, be farther distant from the Center than the Line CD; that is to say, let the perpendicular EH be greater than the Perpendicular EF: I say that CD is greater than GI. Draw the Lines EC, EG.

Demonstration. The Squares CF, FE, (by the 47th of the 1st) are equal to the Square of CE: Now CE is equal to EG, and the Square of EG is equal to the Squares of GH, HE: By consequence the Squares of CF, FE, are equal to the Squares of GH, HE, and taking away on the one Side the Square of HE, and on the other the Square of FE, which

which is less, the Square of CF shall be greater than the Square of GH. Thence the Line CF shall be greater than GH; and the whole Line CD, the double of CF shall be greater than GI, the double of GH.

U S E.

Theodofius maketh use of those Two last Propositions to Demonstrate, that in the Sphere, the least Circles are farthest distant from the Center. We make use thereof in Astrolabes. Moreover to those Propositions might be brought the question of Mechanicks, proposed by Aristotle, which certifies that the Rowers which are in the middle of a Galley, has a greater strength than those which are a stern or a head, because that the sides of the Galley being rounded, have the Oars which be there placed longest. Also the Demonstration of the Rainbow in the Heavens supposeth this Proposition.

PROPOSITION XVI.
THEOREM.

The Perpendicular Line drawn to the extrem part of the Diameter is wholly without the Circle, and toucheth the same. All other Lines drawn between it, and the Circumference of the Circle cut it, and go within the same.

G

Draw

Draw by the Point A, which is the extremity of the Diameter AB, the Perpendicular AC: I say first, that all other Points of the same Line, as the Point C, are without the Circle. Draw the Line DC.

Demonstration. Because the Angle DAC of the Triangle DAC, is a Right Angle; DCA shall be acute, and (by the 19th of the first) the Side DC shall be greater than the Side DA; thence the Line DC passeth beyond the Circumference of the Circle.

I farther add that the Line CA toucheth the Circle, because in the meeting thereof in the Point A, it cutteth it not, but hath all its other Points without the Circle.

I say also that there cannot be drawn any other Line from the Point A, underneath CA, which shall not cut the Circle; as for example, the Line EA. Draw the Perpendicular DI.

Demonstration. Seeing the Angle IAB is Acute, the Perpendicular drawn from the Point D shall be on the same Side with the Angle IAB (by the 17th of the 1st.) Let it be DI; the Angle DIA is Right, and the Angle IAD Acute; AD shall be greater than DI; thence the Line DI, doth not reach the Circumference,

ference, and the Point I is within the Circle.

PROPOSITION XVII.

PROBLEM.

From a Point taken without the Circle, to draw a Line to touch the same.

To draw a touch Line, from the Point A to the Circle BD; draw the Line AC to its Center; and at the Point B, the Perpendicular BE; which cutteth in E a Circle described on the Center C, passing through the Point A. Draw also the Line CE. I say that the Line AD toucheth the Circle in the Point D.

Demonstration. The Triangle EBC, ADC, have the same Angle C; and the Sides CD, CB; CA, CE, equal, by the Definition of a Circle: so they are equal in every respect (by the 4th of the 1st:) and the Angles CBE, CDA are equal: Now CBA is Right; and (by the 16th,) the Line AD shall touch the Circle.

PROPOSITION XVIII.

THEOREM.

The Line drawn from the Center of a Circle to the Point where a straight Line toucheth the same, is Perpendicular to the same Line.

G 2.

If

If there be drawn the Line CD, from the Center C, to the Point of touching D: CD shall be Perpendicular to AB. For if it be not; draw BC Perpendicular to AB.

Demonstration. Since we would have it that the Line CB be Perpendicular; the Angle B shall be Right, and CDB Acute, (*by the 32d of the 1st.*) Thence the Line CB, opposite to the lesser Angle, shall be lesser than CD; which is impossible, seeing CF is equal to CD.

PROPOSITION XIX.

THEOREM.

The Perpendicular drawn to a touch Line at the Point of touching Passeth through the Center.

Let the Line AB touch the Circle in the Point D; and let the Line DC be Perpendicular to AB: I say that DC passeth through the Center. For if it passeth not through it, drawing from the Center to the Point D a Line; it would be Perpendicular to AB: and so Two Lines Perpendicular to the same, would be drawn to the same Point D; which cannot be.

USE.

THe use of touch Lines is very common in Trigonometry, which hath obliged us to make a Table thereof, it serveth us to measure all sorts of Triangles, even Spherical. We have in Opticks some Propositions founded on these touch Lines, as when we are to determine the part of a Globe which is enlightened. The Theoric of the Phases of the Moon is established on the same Doctrine, as also that Celebrated Problem by which Hipparchus findeth the distance of the Sun, by the difference of the true and apparent Quadratures. In the Gnomonicks, the Italian and Babylonian hours, are often Tangent Lines. We measure the Earth by a Line which toucheth it's Surface. And we take in the Art of Navigation, a touch Line to be our Horizon.

PROPOSITION XX.

THEOREM.

THe Angle of the Center is double to an Angle of the Circumference, which hath the same Arch for Base.

If the Angle ABC which is in the Center, and the Angle ADC which is in the Circumference, have the same Arch AC for Base; the first shall be double

double to the second, this Proposition hath three Cases; the first is when the Line ABD passeth through the Center B.

Demonstration. The Angle ABC is exterior in respect of the Triangle BDC: Thence (by the 32^d of the 1st) it shall be equal to the Two Angles D, and C; which being equal (by the 5th of the 1st) because the Sides BC, BD, are equal; the Angle ABC shall be double of each.

The Second case is, when an Angle encloseth the other, and the Lines making the same Angles, not meeting each other, as you see in the second figure; the Angle BID is in the Center, and the Angle BAD is at the Circumference. Draw the Line AIC through the Center.

Demonstration. The Angle BIC is double to the Angle BAC; and CID, double to the Angle CAD, (by the preceding case.) Therefore the Angle BID shall be double to the Angle BAD.

U S E.

T Here is given ordinarily a practical way to describe a Horizontal Dial, by a single opening of the Compass, which is grounded in part on this Proposition. Secondly, when we would determine the Apogee of the Sun, and the
 excen-

excentricity of his Circle, by three Observations ; we suppose that the Angle at the Center is double to the Angle at the Circumference. Ptolomy makes use of this Proposition, to determine as well the excentricity of the Sun, as the Moon's Epicycle. The first Proposition of the Third Book of Trigonometry is grounded on this.

PROPOSITION XXI. THEOREM.

THe Angles which are in the same Segment of a Circle; or that have the same Arch for Base, are equal.

If the Angles BAC, BDC, are in the same Segment of a Circle, greater than a Semicircle ; they shall be equal, Draw the Lines BI, CI.

Demonstration. The Angles A and D are each of them half of the Angle BIC, by the precedeing Proposition ; therefore they are equal. They have also the same Arch BC for Base.

Secondly, Let the Angles A and D be in a Segment BAC, less than a Semicircle ; they shall notwithstanding be equal.

Demonstration. The Angles of the Triangle ABE are equal to all the Angles of the Triangle DEC : The Angles

gles ECD, ABE, are equal (*by the pre-
ceding Case,*) since they are in the same
Segment ABCD, greater than a Semi-
circle: the Angles in E are likewise e-
qual (*by the 15th of the 1st*) therefore
the Angles A and D shall be equal,
which Angles have also the same Arch
BFC, for Base.

U S E.

Prop. XXI. **I**T is proved in Opticks, that the Line
BC shall appear equal, being seen
from A and D; since it always appear-
eth under equal Angles.

We make use of this Proposition to
describe a great Circle, without having
its Center. For Example, when we
would give a Spherical Figure, to Brass
Cauldrons to the end we may work there-
on; and to polish Prospective or
Telescope-Glasses. For having made in
Iron an Angle BAC equal to that which
the Segment ABC contains; and having
put in the Points B, and C, two small
Pins of Iron, if the Triangle BAC be
made to move after such a manner, that
the Side AB may always touch the Pin
B; and the Side AC, the Pin C: The
point A shall be always in the Circumfe-
rence of the Circle ABCD. This way
of describing a Circle may also serve to
make large Astrolabes. PRO:

PROPOSITION XXII.

THEOREM.

Quadrilateral Figures described in a Circle, have their opposite Angles equal to two Right.

Let a Quadrilateral or four sided Figure, be described in a Circle; in such manner that all its Angles touch the Circumference of the Circle ABCD: I say that its opposite Angles BAD, BCD, are equal to two Right. Draw the Diagonals AC, BD.

Demonstration. All the Angles of the Triangle BAD are equal to two Right. In the place of its Angle ABD, put the Angle ACD, which is equal thereto (by the 21st) as being in the same Segment ABCD; and in the place of its Angle ADB, put the Angle ACB, which is in the same Segment of the Circle BCDA. So then the Angle BAD, and the Angles ACD, ACB, that is to say, the whole Angle BCD, is equal to two Right.

U S E

Ptolomy maketh use of this Proposition, to make the Tables of Chords or Subtendents. I have also made use thereof in Trigonometry, in the Third Book.

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Book, to prove that the Sides of an Ob-
tuse Angled Triangle, hath the same
reason amongst themselves, as the Sines
of their opposite Angles.

PROPOSITION XXIII.

THEOREM.

TWo like Segments of Circles de-
scribed on the same Line, are
equal.

I call like Segments of a Circle, those
which contain equal Angles; and I say
that if they be described on the same
Line AB, they shall fall one on the
other, and shall not surpass each other
in any part. For if they did surpass
each other, as doth the Segment ACB,
the Segment ADB, they would not
be like. And to demonstrate it, draw
the Lines ADC, DB, and BC.

Demonstration. The Angle ADB is
exterior in respect of the Triangle
BDC. Thence (by the 21st of the 1st)
it is greater than the Angle ACB, and
by consequence, the Segments ADB,
ACB, containeth unequal Angles,
which I call unlike.

PROPOSITION XXIV.

THEOREM.

TWo like Segments of a Circle de-
scribed on equal Lines, are equal.

If the Segments of Circles AEB, GED, are like, and if the Lines AB, CD, are equal, they shall be equal.

Demonstration. Let it be imagined that the Line CD be placed on AB, they shall not surpass each other, seeing they are supposed equal; and then the Segments AEB, CED, shall be described on the same Line; and they shall then be equal by the preceding Proposition.

U S E.

Curved Lined Figures are often reduced to Right Lined by this Proposition. *Use 24.* As if one should describe two like Segments of Circles, AEC, ADB, on the equal sides AB, AC, of the Triangle ABC: It is evident that Transposing the Segment AEC, on ADB, the Triangle ABC is equal to the Figure ADBCEA.

PROPOSITION XXV.

PROBLEM.

TO compleat a Circle whereof we have but a part.

There is given the Arch ABC, and we would compleat the Circle: There needeth but to find its Center. Draw the Lines AB, BC, and having divided them

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them in the middle in D and E ; draw their Perpendiculars DI, EI, which shall meet each other in the Point I the Center of the Circle.

Demonstration. The Center is in the Line DI (by the Coroll. of the 1st.) It is also in EI ; it is then in the Point I.

U S E.

Uſe 25.

THis Proposition cometh very frequently in use ; it might be propounded another way ; as to inscribe a Triangle in a Circle ; or to make a Circle pass through three given Points, provided they be not placed in a straight Line. Let be proposed the points A, B, C ; put the point of the Compass in A, and at what opening soever describe two Arks F and E. Transport the foot of the Compass to B ; and with the same opening, describe two Arks to cut the former in E and F. Describe on B as Center, at what opening soever, the Arches H and G ; and at the same opening of the Compass, describe on the Center A two Arks, to cut the same in G and H. Draw the Lines FE, and GH, which will cut each other in the point D the Center of the Circle. The Demonstration is evident enough ; for if you had drawn the Lines AB, EC, you would have

have divided them equally, and perpendicularly by so doing. This is very necessary to describe Astrolabes, and to compleat Circles, of which we have put a part. That Astronomical Proposition which teacheth to find the Apogee, and the excentricity of the Sun's Circle, requires this Proposition. We often make use thereof in the Treatise of cutting of Stones.

PROPOSITION XXVI.
THEOREM.

THe equal Angles which are at the Center, or at the Circumference of equal Circles, have for Base equal Arks.

If the equal Angles D and I are in the Center of equal Circles ABC , EFG ; the Arks BC , FG , shall be equal. For if the Ark BC was greater or lesser than the Ark FG : seeing that the Arks are the measure of the Angles; the Angle D would be greater or lesser than the Angle I .

And if the equal Angles A and E be in the Circumference of the equal Circles; the Angles D and I which are the double of the Angle A and E , being also equal; the Arks BC , FG , shall be also equal.

PRO.

PROPOSITION XXVII.

THEOREM.

The Angles which are either in the Center, or in the Circumference of equal Circles, and which have equal Arks for Base, are also equal.

If the Angles D and I are in the Centers of equal Circles; and if they have for Base equal Arks BC, FG, they shall be equal, because that their measures BC, FG, are equal; and if the Angles A and B be in the Circumference of equal Circles, have for Base equal Arks BC, FG, the Angles in the Center shall be equal; and they being their halves (by the 20th.) shall be also equal.

PROPOSITION XXVIII.

THEOREM.

Equal Lines in equal Circles, correspond to equal Arks.

If the Line BC, EF, are applied in equal Circles, ABC, DEF, they shall be Chords of equal Arks, BC, EF. Draw the Lines AB, AC, DE, EF.

Demonstration. In the Triangles ABC, DEF, the Sides AB, AC, DE, EF, are equal, being the Semi-Diameters of equal Circles; the Bases BC, EF,

are supposed equal: therefore (by the 8th of the 1st) the Angles A and D shall be equal; and (by the 16th) the Sides BC , EF shall be also equal.

PROPOSITION XXIX.

THEOREM.

Lines which subtend equal Arks, in equal Circles, are equal.
If the Lines BC , EF , subtend equal Arks BC , EF , in equal Circles, those Lines are equal.

Demonstration. The Arks BC , EF , are equal, and parts of equal Circles: therefore (by the 17th) the Angles A and D shall be equal. So then in the Triangles GAB , HDF , the Sides AB , AC , DE , DF , being equal, as also the Angles A and D , the Bases BC , EF , shall be equal (by the 4th of the 1st).

USE.

Theodosius demonstrates by the 18th, and 19th, that the Arks of the Circles of the Italian and Babylonian hours comprehended between Two Parallels are equal. We demonstrate also after the same manner, that the Arks of Circles of Astronomical hours, comprehended between Two Parallels to the Equator, are equal: these Propositions come

some things continually in use in spherical Trigonometry, as also in Gnomonicks.

PROPOSITION XXX.

PROBLEM.

TO divide an Ark of a Circle into Two equal parts.

It is proposed to Divide the Ark AEB into Two equal parts, put the Foot of the Compass in the Point A, make Two Arks F and G; then transporting the Compass without opening or shutting it, to the Point B, describe two Arks cutting the former in F and G, the Line GF, will cut the Ark AB equally in the Point E. Draw the Line AB.

Demonstration. You divide the Line AB equally by the construction, for imagine the Lines AE, BF, AG, BG, which I have not drawn, lest I should imbroil the figure, the Triangles FGA, FGB, have all their Sides equal; so then (by the 8th of the 1st) the Angles AFD, BFD, are equal. Moreover the Triangles DFA, DFB, have the Sides DF, common; the Sides AF, BF, equal, and the Angles DFA, DFB, equal; whence (by the 4th of the 1st) the Bases AD, DB, are equal, and the

An-

Angles ADF, BDF, are equal. We have then divided the Line AB equally and perpendicularly in the Point D. So then (*by the 1st*) the Center of the Circle is in the Line EG. Let it be the Point C, and let be drawn the Lines CA, CB, all the Sides of the Triangles ACD, BCD, are equal: Thence the Angles ACD, BCD, are equal (*by the 8th of the 1st*;) and (*by the 27th*;) the Arks AE, EB, are equal.

U S E.

AS we have often need to divide an Ark in the middle, the practice of this Proposition is very ordinarily in use, it is by this means we divide the Mariners Compass into 32 Rumbs: for having drawn Two Diameters, which cut each other at Right Angles; we divide the Circle in Four, and sub-dividing each quarter in the middle, we have Eight parts, and sub-dividing each part twice, we come to Thirty Two parts. We have also occasion of the same practice, to divide a Semicircle into 180 degrees; and because for the performing the same Division throughout; we are obliged to divide an Ark into Three, all the Ancient Geometricians have endeavoured to find a method to divide an Angle

Angle or an Ark into Three equal parts, but it is not yet found.

PROPOSITION XXXI.
THEOREM.

THE Angle which is in a Semi-circle is Right, that which is comprehended in a greater Segment, is Acute; and that in a lesser Segment, is Obtuse.

If the Angle BAC be in a Semicircle: I demonstrate that it is Right. Draw the Line DA.

Demonstration. The Angle ADB exterior in respect of the Triangle DAC is equal (*by the 32d. of the 1st*) to the Two Interior DAC, DCA; and those being equal (*by the 5th of the 1st*) seeing the Sides DA, DC, are equal; it shall be double to the Angle DAC. In like manner the Angle ADC is double to the Angle DAB: therefore the Two Angles ADB, ADC, which are equal to Two Right, are double to the Angle BAC, and by consequence the Angle BAC is a Right Angle.

Secondly, the Angle AEC which is in the Segment AEC, is obtuse; for in the Quadrilateral ABCE, the Opposite Angles E and B, are equal to Two Right (*by the 22d*) the Angle B is Acute; therefore the Angle E shall be Obtuse.

Thirdly

Thirdly, the Angle B which is in the Segment ABC, greater than a Semicircle, is Acute; seeing that in the Triangle ABC, the Angle BAC is a Right Angle.

U. S. E.

THE Workmen have drawn from Usc 31. this Proposition the way of trying if their Squares be exact; for having drawn a Semicircle BAD, they apply the Point A of their Square BAD, on the Circumference of the Circle, and one of its Sides AB, on the Point B of the Diameter; the other Side AD must touch the other Point D, which is the other end of the Diameter.

Ptolomy makes use of this Proposition to make the Table of Subtendents or Chords, of which he hath occasion in Trigonometry.

We have also a practical way to erect Usc 31. a perpendicular on the end of a Line, which is founded on this Proposition. For Example, to erect a Perpendicular from the Point A of the Line AB, I put the Foot of the Compass on the Point C, taken at discretion; and extending the other to A, I describe a Circle which may cut the Line AB in the Point B. I draw the Line BCD. It is evident that the
Line

Line DA shall be perpendicular to the Line AB; seeing the Angle BAD is in the Semi circle.

PROPOSITION XXXII.

THEOREM.

The Line which cutteth the Circle at the Point of touching, maketh with the touch Line the Angles equal to those of the Alternate Segments.

Let the Line BD cut the Circle in the Point B, which is the Point where the Line AB doth touch the same: I say that the Angle CBD which the Line BD comprehendeth with the touch Line ABD, is equal to the Angle E, which is that of the Alternate Segment BED; and that the Angle ABD, is equal to the Angle F, of the Segment BFD.

In the first place, if the Line passeth through the Center, as doth the Line BE: It would make with the touch Line AB, two Right Angles (*by the 17th*) and the Angle of the Semi-circles would be also Right (*by the preceding.*) So the Proposition would be true.

If the Line passeth not through the Center, as doth the Line BD: Draw the Line BE through the Center, and joyn the Line DE.

Demon-

Demonstration. The Line BE maketh two Right Angles with the touch Line, and all the Angles of the Triangle BDE are equal to Two Right (by the 32d of the 1st.) So taking away the Right Angles ABE, and D, which is in a Semi-circle, and taking again away the Angle EBD, which is common to both, the Angle CBD shall be equal to the Angle BED.

Thirdly, the Angle ABD is equal to the Angle F, because in the Quadrilateral BFDE, which is inscribed in a Circle, the opposite Angles E, and F, are equal to two Right (by the 22d) the Angles ABD, DBC, are also equal to Two Right by the 13th of the 1st) and the Angle DBC, and E, are equal, as just now I did demonstrate; therefore the Angles ABD, and BFD, are equal.

U S E.

THis Proposition is necessary for that which followeth.

PROPOSITION XXXIII.

PROBLEM.

TO describe upon a Line, a Segment of a Circle, which shall contain a given Angle.

It

It is proposed to describe on the Line AB, a Segment of a Circle to contain the Angle C. Make the Angle BAD equal to the Angle C, and draw to AD the Perpendicular AE. Make also the Angle ABF, equal to the Angle BAF; and lastly describe a Circle on the Point F, as Center, at the opening BE, or FA, the Segment BEA containeth an Angle equal to the Angle C.

Demonstration. The Angles BAF, ABF, being equal, the Lines FA, FB, are equal (*by the 6th*) and the Circle which is described on the Center F passeth through A and B. Now the Angle DAE being Right, the Line DA toucheth the Circle in the Point A (*by the 16th.*) therefore the Angle which the Segment BEA comprehended as the Angle E, is equal to the Angle DAB; that is to say, to the Angle C.

But if the Angle was obtuse; we must take the Acute Angle which is its Complement to 180 degrees.

PROPOSITION XXXIV.

PROBLEM.

A Circle being given, to cut therefrom a Segment to contain an assigned Angle.

To

To cut from the Circle BCE, a Segment to contain the Angle A. Draw (by the 17th) the touch Line BD; and make the Angle DBC equal to the Angle A. It is evident (by the 32d) that the Segment BEC contains an Angle equal to DBC, and by consequence to the Angle A.

USE.

I Have made use of this Proposition to find Geometrically the excentricity of the Annual Circle of the Sun, and his Apogæon, three Observations being given. It is also made use of in Opticks, two unequal Lines being proposed, to find a point where they shall appear equal, or under equal Angles; making on each, Segments which may contain equal Angles.

PROPOSITION XXXV.
THEOREM.

IF two Lines cut each other in a Circle, the Rectangle comprehended under the parts of the one, is equal to the Rectangle comprehended under the parts of the other.

In the first place, If two Lines cut each other in the Center of the Circle, they shall be equal and divided equally,
so

so then it is evident, that the Rectangle comprehended under the parts of the one, is equal to the Rectangle comprehended under the parts of the other.

Secondly, let one of the Lines pass through the Center F; as AC, and divide the Line BD in two equally in the Point E; I say that the Rectangle comprehended under AE, EC, is equal to the Rectangle comprehended under BE, ED, that is to say to the Square of BE. The Line AC is perpendicular to BD (*by the 3d.*)

Demonstration. Seeing that the Line AC is divided equally in F, and unequally in E, the Rectangle comprehended under AE, EC, with the Square of FE, is equal to the Square of FC, or FB, (*by the 5th of the 2d.*) Now the Angle E being Right, the Square of FB is equal to the Squares of BE, EF; therefore the Rectangle comprehended under AE, EC, with the Square of EF, is equal to the Squares of BE, EF; and taking away the Square of EF, there remains that the Square of BE, is equal to the Rectangle under BE, ED.

Thirdly, let the Line AB pass through the Center F, and let it divide the Line CD unequally in the Point E: Draw
FG

FG perpendicularly to CD; and (by the 3d) the Lines CG, GD, shall be equal.

Demonstration. Seeing the Line AB is divided equally in F, and unequally in E; the Rectangle comprehended under AE, EB, with the Square of EF, is equal to the Square of BF, or FC, (by the 5th of the 2d.) In the place of EF, put the Squares of FG, GE, which is equal thereto, (by the 47th of the 1st.) In like manner the Line CD being equally divided in G, and unequally in E: the Rectangle CED, with the Square of GE, shall be equal to the Square of GC. Add the Square of GF; the Rectangle of CE, ED, with the Squares of GE, FG, shall be equal to the Squares of GC, GF; that is to say (by the 47th of the 1st) to the Square of CF. Therefore the Rectangle AEB, with the Squares of GE, GF; and the Rectangle of CE, ED, with the same Squares are equal; and by consequence taking away the same Squares, the Rectangle AEB is equal to the Rectangle CFD.

Fourthly, let the Lines CD, HI, cut each other in the Point E, so that neither of them pass through the Center. I say that the Rectangle CED is equal

H

to

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to the Rectangle HEL . For drawing the Line AFB , the Rectangles CED , HEL , are equal to the Rectangle ABB , (*by the preceding Case.*) therefore they are equal.

U S E.

One might by this Proposition have a practical way to find the Fourth proportional to three given Lines, or the third proportional to two Lines.

PROPOSITION XXXVI.
THEOREM.

IF from a Point taken without a Circle, one draw a touch Line, and another Line to cut the Circle, the Square of the Tangent Line shall be equal to the Rectangle comprehended under the whole Secant, and under the exterior Line.

Let from the Point A , taken without the Circle, be drawn a Line AB , to touch the same in B ; another AC , or AH , to cut the Circle. The Square of AB shall be equal to the Rectangle comprehended under AC , AO ; as also to the Rectangle comprehended under AH , AF . If the Center pass through the Center, as AC ; draw the Line EB .

Demonstration. Seeing the Line OC , is divided in the middle in E , and that thereto

thereto is added the Line AO; the Rectangle comprehended under AO, AC, with the Square of OE, or EB, shall be equal to the Square of AE (*by the 6th of the 2d.*) Now the Line AB toucheth the Circle in the Point B: so (*by the 17th*) the Angle B is Right; and (*by the 47th of the 1st*) the Square of AE is equal to the Squares of AB, EB. Therefore the Rectangle under AC, AO, with the Square of EB, is equal to the Squares of AB, EB: and taking away the Squares of EB; the Rectangle under AC, AO, shall be equal to the Square of AB.

Secondly, let the Secant AH not pass through the Center. Draw the Line AH, the perpendicular EG, which shall divide in the middle in G, the Line FH: draw also the Line EF.

Demonstration. The Line FH being divided equally in the point G; and the Line AF being added thereto; the Rectangle comprehended under AH, AF, with the Square of FG, shall be equal to the Square of AG. Add to both the Square of EG; the Rectangle AH, AF, with the Square of FG, GE, that is to say (*by the 47th of the 1st*) with the Square of FE or EB, shall be equal to the Square of AG, GE, that is to say

(by the 47th of the 1st) to the Square of AE. Moreover, the Square of AE, (by the same) is equal to the Square of EB, AB: therefore the Rectangle comprehended under AH, AF, with the Square of BE, is equal to the Squares of BE, AB, and taking from both the Square of BE, the Rectangle comprehended under AH, AF, shall be equal to the Square of AB.

Coroll. 1. If you draw several Secants AC, AH; the Rectangles AC, AO; and AH, AF, shall be equal amongst themselves, seeing that the one and the other are equal to the Square of AB.

Coroll. 2. If there be drawn two Tangents AB, AI, they shall be equal, because their Squares are equal to the same Rectangle AC, AO; and by consequence amongst themselves; as also the Lines.

PROPOSITION XXXVII.

THEOREM.

IF the Rectangle comprehended under the Secant, and under the exterior Line, is equal to a Line which falls on the Circle; that Line toucheth the same.

Let be drawn the Secant AC, or AH, and let the Rectangle AC, AO, or the Rectangle AH, AF, be equal to the Square

Square of the Line AB; that Line shall touch the Circle: draw the touch Line AI (by the 17th) and the Line IE.

Demonstration. Seeing the Line AI toucheth the Circle, the Rectangle AC, AO; or AH, AF, shall be equal to the Square of AI. Now the Square of AB, is supposed equal to each Rectangle: therefore the Squares of AI, and of AB, are equal; and by consequence the Lines AI, AB. So then the Triangles ABE, AEL, which have all their Sides equal to each other, shall be equiangular (by the 8th of the 1st) and seeing the Angle AIE is Right (by the 17th) the Line AI being a touch Line, the Angle ABE shall be Right, and the Line AB a touch Line also (by the 16th.)

U S E.

MAurolicus maketh use of this Proposition to find the Diameter of the Earth. For looking from the top of a Mountain OA, to the extremity of the Earth, along the Line BA; he observeth the Angle OAB, which the Line AB, maketh with a plumb Line AC: and he concludes the Length of the Line AB, by a Trigonometrical Calculation. He Multiplieth AB by AB to have its Square, which he divides by AO, the height of

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the Mountain: from which having taken
away AO there remains OC, the Diame-
ter of the Earth, this Proposition serveth
also to prove the Fifth Proposition of the
Third Book of Trigonometry.*

The end of the Third Book.

THE
FOURTH BOOK
OF
Euclid's *Elements*:

THis Book is very useful in
Trigonometry, seeing that
by inscribing of Polygons in a
Circle, we have the practice of
making the Table of Subtendants, of Sines,
of Tangents, and of Secants; which are
very necessary in all sorts of measuring.
Secondly, in inscribing of Polygons in
a Circle, we have the diversity of the
Aspects of the Planets, which take their
Names from the same Polygons.

Third.

Thirdly, in the practice hereof we have a method which giveth the Quadrature of the Circle, as near the truth as shall be needful. We demonstrate also that a Circle is in duplicate reason to its Diameter.

Fourthly, Military Architecture bath need of Inscribing Polygons into Circles, in the designing or drawing of regular Fortifications.

The DEFINITIONS.

1. **A** Right Lined figure is inscribed Plate V.
in a Circle; or the Circle is de- Fig. 1.
scribed about a figure, when all its Angles are in the Circumference of the same Circle. As the Triangle ABC is inscribed in a Circle, and the Circle is described about the Triangle, because the Angles A, B, C, do touch the Circumference, the Triangle DEF is not inscribed in the Circle, because the Angle D, doth not touch the Circumference of the Circle.

2. A Right Lined Figure is described Fig. 1.
about a Circle, and the Circle is inscribed within the same figure, when all the sides of the figure touch the Circumference of the Circle. As the Triangle GHI, is described about the Circle KLM, because its Sides touch the Circle in K, L, M. H 4 3. A

3. A Line is fitted or inscribed in a Circle, when it is terminated at both ends by the Circumference of the Circle: *As the Line NO. The Line RP is not inscribed in the Circle.*

PROPOSITION I.
PROBLEM.

TO Inscribe in a Circle a Line, which surpasseth not its Diameter.

It is proposed to inscribe in a Circle AEBD, a Line which surpasseth not its Diameter, take the length thereof on the Diameter, and let it be for Example BC. Put the Foot of the Compass in B, and describe a Circle at the extent of BC, which cutteth the Circle AEBD in D, and E. Draw the Line BD, or BE. It is evident they are equal to BC, by the Definition of a Circle.

USE.

THis Proposition is necessary for the practice of those which follow.

PROPOSITION II.
PROBLEM.

TO Inscribe in a Circle, a Triangle equiangled to another Triangle.

The Circle EGH is proposed, in which one would inscribe a Triangle equi-

equiangled to the Triangle ABC. Draw the touch Line FED, (by the 17th of the 3d) and make at the Point of touching E, the Angle DEH equal to the Angle B; and the Angle FEG, equal to the Angle C (by the 23d of the 1st.) Draw the Line GH, the Triangle EGH shall be equi-angled to ABC.

Demonstration. The Angle DEH is equal to the Angle EGH, of the Alternate Segment (by the 32d of the 3d) now the Angle DEH was made equal to the Angle B, and by consequence the Angles B and G are equal. The Angles C and H are also equal, for the same reason; and (by the Coroll. 2. of the 32d of the 1st) the Angles A and GEH, shall be equal. Therefore the Triangles EGH, ABC, are equiangled.

PROPOSITION HL

PROBLEM.

TO describe about a Circle, a Triangle equiangled to another.

If one would describe about a Circle GKH, a Triangle equiangled to ABC; one of the Sides BC, must be continued to D and E; and make the Angle GH equal to the Angle ABD: and HK equal to the Angle ACE, then draw the Tangents LGM, LKN, NHM, through

through the Points G, K, H . The Tangents shall meet each other; for the Angles IKL, IGL , being Right, if one should draw the Line KO , which is not drawn; the Angles KGL, GKL , would be less than two Right: therefore (by the 11th Axiom) the Lines GL, KL , ought to co-incide.

Demonstration. All the Angles of the Quadrilateral $GHMN$, are equal to four Right; seeing it may be reduced into two Triangles; the Angles IGM, IHM , which are made by the Tangents, are Right; thence the Angles M and I are equivalent to two Right as well as the Angles ABC, ABD . Now the Angle GLH , is equal to the Angle ABD by construction: therefore the Angle M shall be equal to the Angle ABC . For the same reason the Angles N and ACB are equal; and so the Triangles LMN, ABC , are equiangled.

PROPOSITION IV.

PROBLEM.

To inscribe a Circle in a Triangle.

IF you would inscribe a Circle in a Triangle ABC : divide into two equally the Angles ABC, ACB , (by the 9th of the 1st) drawing the Lines CD ,

CD, BD, which concur in the Point D. Then draw from the Point D, the Perpendiculars DE, DF, DG, which shall be equal; so that the Circle described on the point D, at the opening DE, shall pass through F and G.

Demonstration. The Triangles DEB, DBF, have the *Angles* DEB, DFB, equal, seeing they are Right; the *Angles* DBE, DBF, are also equal, the *Angle* ABC having been divided into two equally; the Side DB is common: therefore (by the 26th of the 1st) these Triangles shall be equal in every respect; and the sides DE, DF, shall be equal. One might demonstrate after the same manner, that the sides DF, DG, are equal. One may therefore describe a Circle which shall pass through the Points E, F, G; and seeing the *Angles* E, F, G, are Right, the Sides AB, AC, BC, touch the Circle, which shall by consequence be inscribed in the Triangle.

PROPOSITION V.

PROBLEM.

To describe a Circle about a Triangle.

IF you would describe a Circle about a Triangle ABC; divide the Sides
AB,

AB, BC, into two equally, in D and E, drawing the Perpendiculars DF, EF, which concur in the Point F. If you describe a Circle on the Center F, at the opening FB, it shall pass through A and C; that is to say, that the Lines FA, FB, FC, are equal.

Demonstration. The Triangles ADF, BDF, have the Side DF, common, and the Sides AD, DB, equal, seeing the Side AB hath been divided equally; and the *Angles* in D are equal, being Right. Thence (by the 4th of the 1st) the Bases AF, BF, are equal; and for the same reason the Bases BF, CF.

U S E.

WE have often need to inscribe a Triangle in a Circle; as in the first Proposition of the Third Book of Trigonometry. This Practice is necessary for to measure the Area of a Triangle, and upon several other occasions.

PROPOSITION VI.

PROBLEM.

To inscribe a Square in a Circle.

TO inscribe a Square in the Circle ABCD; draw to the Diameter AB, the Perpendicular DC, which may pass through the Center E. Draw also the Lines AC, CB, BD, AD; and
 ouy

you will have inscribed in the Circle the Square ACBD.

Demonstration. The Triangles AEC, CEB, have their Sides equal, and the Angles AEC, CEB, equal, being they are Right; therefore the Bases AC, CB, are equal (by the 4th of the 1st.) Moreover, seeing the Sides AE, CE, are equal, and the Angle E being Right, they shall each of them be semi-right, (by the 32d, of the 1st.) So then the Angle ECB is semi-right. And by consequence the Angle ACB shall be Right: It is the same of all the other Angles: therefore the Figure ACDB is a Square.

PROPOSITION VII.

PROBLEM.

To describe a Square about a Circle.

HAVING drawn the two Diameters AB, CD, which cut each other perpendicularly in the Center E: draw the touch Lines FG, GH, HI, FI, through the Points A, D, B, C; and you will have described a Square FGHI, about the Circle ACBD.

Demonstration. The Angles E and A are Right; thence (by the 29th of the 1st) the Lines FG, CD are Parallels. I prove after the same manner, that CD, HI; FI, AB; AB, GH, are parallels; thence

hence the Figure $PCDG$, is a parallelogram: and (by the 34th of the 1st) the Lines FG , CD , are equal; as also GD , HC ; FI , AB ; AI , GH : and by consequence the Sides of the Figure PG , GH , HI , IF , are equal. Moreover, seeing the Lines FG , CD , are parallel, and that the Angle FCE is Right; the Angle G shall be also Right, (by the 29th of the 1st.) I demonstrate after the same manner, that the Angles F , H , and I , are Right. Therefore the Figure $FGHI$ is a Square, and its Sides touch the Circle.

PROPOSITION VIII. PROBLEM.

To inscribe a Circle in a Square.

IF you will inscribe a Circle in the Square $FGHI$, divide the Sides FG , GH , HI , FI , in the middle in A , D , B , C ; and draw the Lines AB , CD , which cutteth each other in the Point E . I demonstrate that the Lines EA , ED , EC , EB , are equal; and that the Angles in A , B , C , D , are Right; and that so you may describe a Circle on the Center E , which shall pass through A , D , B , C , and which toucheth the Sides of the Square.

De-

Demonstration. Joining the Lines AB , CH , to make the Line AC , BH , which are parallel, and equal; they shall be also parallel and equal; therefore the Figure $ACBH$ is a parallelogram, and the Lines AE , GD , AG , ED , being parallel; and AG , GD , being equal; AE , ED , shall be also equal. It is the same with the others AF , EC , EB . Moreover AG , ED , being parallels; and the Angle G being Right, the Angle D shall be also a Right Angle. One may then on the Center E , describe the Circle $ADBC$, which shall pass through the Points A , D , B , C , and which shall touch the Sides of the Square.

PROPOSITION IX. PROBLEM.

To describe a Circle about a Square.

TO describe a Circle about a Square AB , FD ; draw the Diagonals AF , BD , which cutteth each other in the Point E . This Point E shall be the Center of the Circle, which will pass through the Points A , F , B , D . I must then demonstrate that the Lines AE , FE , BE , DE , are equal.

Demonstration. The Sides AB , FB ,
are

are equal, and the Angle B is Right: Therefore the Angles EAB, BEA, are equal (by the 5th of the 1st) and semi-right; (by the 22d. of the 1st.) I Demonstrate after the same manner, that the Angles ABD, ADB, FDB, DBF, are Semi-Right. So the Triangle AEB, having the Angles EAB, BEA, semi-right, and consequently equal, it shall also have (by the 26th. of the 1st) the Sides AE, EB, equal. One might Demonstrate after the same manner, that the Lines EF, EB, EF, ED, are equal.

U S E.

WE Demonstrate in the Twelfth Book that Polygons described in Circles, degenerate into Circles, and as those Polygons are always in a duplicate Ratio of their Diameters, that Circles are so likewise. We have also need in Practical Geometry to inscribe a Square and other Polygons, within and without a Circle, to reduce the Circle into a Square.

PROPOSITION X.

PROBLEM.

TO make an Isosceles Triangle which hath each of the Angles on the Base double to the Third.

To.

To make the Isosceles Triangles ABD, which may have each of its Angle ABD, ADB, double to the Angle A, divide the Line AB (by the 11th. of the 2d) in such manner that the Square AC be equal to the Rectangle AB, BC. Describe on the Center A, at the opening AB, a Circle BD; which you shall inscribe BD equal to AC. Draw the Line DC, and describe a Circle about the Triangle ACD, (by the 5th.)

Demonstration. Seeing that the Square of CA, or BD, is equal to the Rectangle comprehended under AB, BC, the Line BD shall touch the Circle ACD, in the Point D, (by the 37th. of the 3d) thence the Angle BDC shall be equal to the Angle A comprehended in the Alternate Segment CAD, (by the 32d. of the Third.) Now the Angle BCD exterior in respect of the Triangle ACD, is equal to the Angle A and CDA; therefore the Angle BCD is equal to the Angle BDA. Moreover the Angle ADB, is equal to the Angle ABD, (by the 5th. of the 1st) thence the Angles BCD, DBC, are equal; and (by the 6th. of the 1st) the Sides BD, DC, shall be equal. And seeing BD is equal to AC, the Sides AC, OD, shall be equal, and the Angles A and CDA shall

shall be so also. Therefore the Angle ADB is double to the Angle A.

PROPOSITION XI.
PROBLEM.

TO inscribe a Regular Pentagon in a Circle.

To inscribe a Regular Pentagon in a Circle; describe (by the 10th) an Isosceles Triangle ABC, which shall have the Angles ABC, ACB, on the Base, each double to the Angle A. Inscribe in the Circle, a Triangle DEF, equi-angled to the Triangle ABC; divide into Two equally the Angles DEF, DFE, draw the Lines EG, FH. Lastly, joyn the Lines DH, DG, GF, EH: and you shall have made a Regular Pentagon, that is to say, which hath all its Sides equal, as well as all its Angles.

Demonstration. The Angles DEG, GEF, DFH, HFE, are the halves of the Angles DEF, DFE, which are each double to the Angle A; and by consequence the five Arches, which serve to them for Bases, are equal (by the 26th. of the 3d) and the Lines HD, HE, EF, FG, GD, are equal (by the 29th of the 3d.) Secondly, the Angles DGE, GFE, having each for Base thereof, the Arcs shall be likewise equal. Thence all the Sides, and

and Angles of the Pentagons are equal.

PROPOSITION XII.

PROBLEM.

TO describe a Pentagon about a Circle.

Inscribe a Regular Pentagon ABCDE in the Circle (by the 11th.) Drawing the Tangents through the Points A, B, C, D, E, (by the 17th of the 3d) you will have described a Regular Pentagon about the Circle. Draw the Lines FA, FG, FE, FH, FD:

Demonstration. The touch Lines GA, GE, are equal (by the Coroll. of the 36th of the 3d) as also EH, HD; the Lines FA, FD, are also equal by the Definition of a Circle: thence (by the 8th of the 1st) the Triangles FGA, FGE, are equal in every respect; and the Angles AFG, EFG, are equal; as also the Angles FEH, DFH. And because that the Angles EFA, EFD, are equal (by the 27th of the 3d) their halves EFH, EFG, shall be equal: and (by the 26th of the 1st) the Triangles FEH, FEG, shall be equal in every respect; and the Sides EG, EH, also equal. I demonstrate after the same manner, that each of the Sides are divided equally into two parts; and by consequence joining the Lines AG, GE, are

are equal; GH, HI, shall be also equal. Moreover, the *Angles* G and H, being double to the *Angles* FGE, FHI, are also equal. We have then described a Regular Pentagon about a Circle.

PROPOSITION XIII. PROBLEM.

TO inscribe a Circle in a Regular Pentagon.

To inscribe a Circle in the Regular Pentagon ABCDE; divide the *Angles* A and B into two equally by the Lines AF, BF, which will meet at the Point F. Then drawing the Line FG perpendicular to AB describe a Circle on the Center F, at the opening FG. I say that it will touch all the other Sides, that is to say, having drawn FH perpendicular to BC; FG and FH shall be equal.

Demonstration. Seeing the equal *Angles* A and B have been divided into two equally, their halves GAF, GBF, shall be equal; and seeing the *Angles* in G are Right, the Triangles AFG, BFG, shall be equal in every respect (by the 26th of the 1st.) So the Lines AG, GB, are equal. Moreover, I prove that the Lines BG, BH; as well as FG, FH, are equal: and the Sides AB, BC,

of

of a Regular Pentagon being equal ; the Lines BH, HC, shall be equal : and by consequence the *Angles* G and H being right and equal, the Triangles BFH, HFC, shall be equal in every respect : and the *Angles* FBH, FCH, shall be equal : and seeing the *Angles* B and C are equal, the *Angle* FCH shall be the half of the *Angle* C. So going from the one to the other, I demonstrate that all the Perpendiculars FG, FH, &c. are equal.

PROPOSITION XIV.

PROBLEM.

TO describe a Circle about a Regular Pentagon.

To describe a Circle about a Regular Pentagon ABCDE : divide two of its Sides AB, BC, in G and H ; draw the Perpendiculars GF, HF. The Circle drawn on the Center F, at the opening FA, shall pass through B, C, D, E.

Demonstration. Suppose that the Circle be described already, it is evident (by the 1st of the 3^d) that having divided the Line AB in the middle in G ; and having drawn the perpendicular GF, the Center of the Circle is in that perpendicular ; it is also in FH, and therefore must be in the Point of their intersection at F. U S E.

USE.

These Propositions are only useful to make the Table of Sines, and for tracing out the ground of Cittadels: for Pentagons are the most ordinary. You must also take notice, that these ways of describing a Pentagon about a Circle, may be applied to the other Polygons. I have given another way to inscribe a Regular Pentagon in a Circle, in Military Architecture.

PROPOSITION XV.
PROBLEM.

TO inscribe a Regular Hexagon in a Circle.

To inscribe a Regular Hexagon in the Circle ABCDEF: draw the Diameter AD, and putting the Foot of the Compass in the Point D, describe a Circle at the opening DG; which shall intersect the Circle in the Points EC; then draw the Diameters EGB, CGF, and the Lines AB, AF, and the others.

Demonstration. It is evident that the Triangles CDG, DGE, are equilateral; wherefore the Angles CGD, DGE, and their opposites BGA, AGF, are each of them the third part of two Right; and that is 60 degrees. Now all the
Angles

Angles which can be made about one Point is equal to four Right; that is to say, 360. So taking away four times 60, that is 240, from 360; there remains 120 degrees; for BGC, and FGE, whence they shall each be 60 degrees. So all the *Angles* at the Center being equal; all the *Arks* and all the *Sides* shall be equal; and each *Angle* A, B, C, &c. shall be composed of two *Angles* of Sixty; that is to say, one hundred and twenty degrees. They shall therefore be equal.

Coroll: The Side of a Hexagon is equal to the Semi-diameter.

U S E.

BEcause that the Side of an Hexagon is the Base of an Ark of Sixty degrees, and that is equal to the Semi-Diameter; its half is the Sine of Thirty; and it is with this Sine we begin the Tables of Sines. Euclid treateth of Hexagons in the last Book of his *Elements*.

PROPOSITION XVI.

P R O B L E M.

TO inscribe a Regular Pentadecagon in a Circle.

Inscribe in a Circle an equilateral Triangle ABC (by the 2d) and a Regular

lar Pentagon (*by the 11th*) in such sort that the *Angles* meet in the Point A. The Lines BF, BL, IE, shall be the Sides of the Pentadecagon: and by inscribing in the other *Arks*, Lines equal to BF, BL, you may compleat this Polygon.

Demonstration. Seeing the Line AB is the Side of the Equilateral Triangle; the *Ark* AEB, shall be the third of the whole Circle, or $\frac{1}{3}$ fifteenths: and the *Ark* AE being the fifth part, it shall contain $\frac{1}{5}$; thence EB contains two: and if you divide it in the middle in I, each part shall be a fifteenth.

U. S. E.

THis Proposition serveth only to open the way for other Polygons. We have in the compass of Proportion, very easy methods to inscribe all the ordinary Polygons; but they are grounded on this: For one could not put Polygons on that Instrument, if one did not find their sides by this Proposition, or such like.

The End of the Fourth Book.

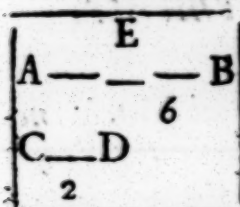
THE
FIFTH BOOK
OF
Euclid's *Elements*.

THis Fifth Book is absolutely necessary to demonstrate the Propositions of the Sixth Book. It containeth a most universal Doctrine, and a way of arguing by Proportion, which is most subtile, solid, and brief. So that all Treatises which are founded on Proportions, cannot be without this Mathematical Logick. Geometry, Arithmetick, Musick, Astronomy, Statics, and to say in one word all the Treatises of the Sciences are demonstrated by the Propositions of this Book. The greatest part of measuring is done by Proportions, and in practical Geometry. And one may demonstrate all the Rules of Arithmetick by the Theorems hereof, wherefore it is not necessary to have recourse to the 7th, 8th, or 9th

I Books.

Books. The Musick of the Ancients is scarce any thing else but the Doctrine of Proportion applied to the Senses. It is the same in Staticks, which considers the Proportion of Weights. In fine one may affirm, that if one should take away from Mathematicians the knowledge of the Propositions that this Book giveth us, the remainder would be of little use.

DEFINITIONS.



A Lesser quantity compared with a greater, is called a part. As if one compare the Line CD of Two Foot, with the Line AB of six, it is called a part. And although in effect CD be not in AB, provided the Line AE equal to CD, is found in AB, there is given thereto the name of part.

The whole corresponds to its part: and this shall be the greater quantity compared with the lesser; whether it contains the same in effect; or that it doth not contain the same.

Parts or quantities taken in general are divided ordinarily into Aliquot parts, and Aliquant parts.

1. An Aliquot part (which Euclid defines

defines in this Book) is a Magnitude of a Magnitude, the lesser of the greater, when it measureth it exactly. *That is to say, that it is a lesser quantity compared with a greater, which it measureth precisely. As the Line of Two Foot taken three times, is equal to a Line of Six Foot.*

2. Multiplex is a Magnitude of a Magnitude, the greater of the lesser, when the lesser measureth the greater: *That is to say, that Multiplex is a great quantity compared with a lesser, which it contains precisely some number of times. For Example, the Line of Six Foot, is treble to a Line of two Foot.*

Aliquant parts, is a lesser quantity compared with a greater, which it measureth not exactly: So a Line of 4 Foot, is an Aliquant part of a Line of 10 Foot.

Equimultiplexes are Magnitudes which contain equally their Aliquot parts, *that is to say the same number of times: For example, if A contains as many*

12.	4.	6.	2.
A	B	C	D

times B, as C contains D; A and C shall be equal Multiplexes of B and D.

3. Reason, (or Ratio) is a mutual habitude or respect of one Magnitude to another of the same Species. I have added of the same Species.

4. For *Euclid* saith, that Magnitudes have the same reason, when being multiplied, they may surpass each other. To do which, they must be of the same Species. In effect, a Line hath no manner of Reason with a Surface, because a Line taken Mathematically is considered without any Breadth, so that if it be multiplied as many times as you please, it giveth no Breadth, and notwithstanding a Surface hath Breadth.

Seeing that Reason is a mutual habitude or respect of a Magnitude to another, it ought to have two terms. That which the Philosophers would call foundation, is named by the Mathematicians Antecedent, and the term is called Consequent. As if we compare the Magnitude A, to the Magnitude B, this habitude or Reason shall have for Antecedent the quantity A, and for consequent the quantity B. As on the contrary, if we compare the Magnitude B, with A, this Reason of B to A, shall have for Antecedent the Magnitude B, and for consequent the Magnitude A.

The Reason or habitude of one Magnitude to another, is divided into rational Reason, and irrational Reason. Rational Reason is a habitude of one Magnitude to another, which is commensurable thereto; that is to say, that those Magnitudes have

a common measure, which measureth both exactly. As the reason of a Line of 4 Foot, to a Line of 6, is rational, because a Line of two Foot measureth both exactly: and when this happeneth, those Magnitudes have the same Reason as one Number hath to another. For Example, because that the Line of two Foot, which is the common measure, is found twice in the Line of 4 Foot, and thrice in the Line of 6; the first to the second shall have the same Reason as 2 to 3.

Irrational Reason is between Two Magnitudes of the same Species which are incommensurable, that is to say, that have not a common measure. As the Reason of the Side of a Square to its Diagonal. For there cannot be found any measure, although never so little, which will measure both precisely.

Four Magnitudes shall be in the same Reason, or shall be Proportionals, when the Reason of the first to the second, shall be the same, or like to that of the third to the fourth: wherefore to speak properly, Proportion is a similitude of Reason. But one findeth it difficult to understand in what consisteth this similitude of Reason. It is only to say that two habits or Relations be alike. For Euclid hath not given a just Definition, and which

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might have explained its Nature ; having contented himself to give us a mark by which we know, if Magnitude have the same reason. And the obscurity of this Definition hath made this Book difficult. I will endeavour to supply this default.

5. Euclid saith, that four Magnitudes have the same reason, when having taken the Equi-multiples of the first, and of the third ; and other Equimultiples of the second, and of the fourth ; whatever combination is made, when the Multiplex of the first is greater than the Multiplex of the second ; the Multiplex of the third shall be also greater than the Multiplex of the fourth : And when the Multiplex of the first is equal, or less than the Multiplex of the second ; the Multiplex of the third is equal or less than the Multiplex of the fourth. That then there is the same reason between the first and second ; as there is between the third and fourth.

A	B	C	D	As if there were proposed four Magnitudes A, B, C, D. Having taken the Equi-multiplexes of A and C, which let be E and G, quintuplex : F and H double to B and D. In like manner, taking K and
2.	4.	3.	6.	
E	F	G	H	
10	8.	15	12	
K	L	M	N	
8.	8.	12	12	
O	P	Q	R	
6.	16	9.	24	

and *M*, quadruple to *A* and *C*: *L* and *N* double to *B* and *D*. Taking again *O* and *Q*, triple to *A* and *C*: *P* and *R* quadruple to *B* and *D*. Now because *E* being greater than *F*, *G* is greater than *H*: and *K* being equal to *L*; *M* is equal to *N*: In fine *O* being lesser than *P*; *Q* is lesser than *R*. Then *A* shall have the same reason to *B*, as *C* to *D*, I believe that Euclid ought to have demonstrated this Proposition, seeing it is so intangled that it cannot pass for a Maxim.

To explain well what Proportion is, it is to say, that four Magnitudes have the same Ratio; altho' one may say in general, that to that end the first must be alike part, or alike whole, in respect of the 2d; as is the 3d, compared to the 4th; notwithstanding, because this definition doth not convene with the reason of equality; there must be given a more general; and to make it intelligible, it must be explained what is meant by a like Aliquot part.

Like Aliquot parts are those which are as many times in their whole, as three in respect of nine; two in respect of six; are alike Aliquot parts, because each are found three times in their whole.

The first quantity will have the same reason to the 2d, as the 3d hath to the 4th, if the first contains as many times any

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Aliquot part of the second whatever, as the 3d. contains alike Aliquot part of the 4th. as if A contains as

A, B, C, D.

many times a Hundreth, a Tousandth, a Millionth part of B: as C contains a Hundreth, a Thousanth, a Millionth part of D; and so of any other Aliquot parts imaginable: there will be the same Reason of A to B; as of C to D.

To make this Definition yet clearer; I will in the first place prove, that if there be the same reason of A to B, as there is of C to D; A will contain as many times the Aliquot parts of B, as C doth of D. And I will afterwards prove, that if A contains as many times the Aliquot parts of B, as C doth of D; there will be the same Reason of A to B, as of C to D.

The first Point seemeth evident enough, provided one doth conceive the terms: for if A contains one Hundred, and one times the tenth part of B; and C only One Hundred times the tenth part of D: the Magnitude A compared with B, would be a greater whole, than C compared with D: so that it could not be compared after the same manner, that is to say, the habitude or Relation would not be the same.

The 2d point seemeth more difficult; to wit, whether if this propriety be so found, the

the Reason shall be the same, that is to say, if AB contains as many times any Aliquot parts whatever of CD, as E contains like Aliquot parts of F: there shall be the same Reason of AB to CD, as of E to F. For I will prove that if there were not the same Reason, A would contain more times any Aliquot part of B, than C containeth alike Aliquot part of D: which would be contrary to what we had supposed.



Let us suppose then that there is a greater reason of AB to CD, than of E to F; that is to say, that AB is

greater than it ought to be, have the same Reason to CD, as E hath to F. Wherefore a lesser Magnitude than AB, for Example, AG shall have the same Reason to CD, as E to F. Divide CD in the middle in H and HD, in the middle in I and ID, in the middle in K. In continuing this Division you would find an Aliquot part of CD less than GB: let it be KD.

Demonstration. Seeing there is the same Reason of AG to CD, as of E to F, AG will contain as many times KD an Aliquot part of CD, as E would contain a like Aliquot part of F. Now AB con-

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tains KD, once more than AG: thence AB will contain once more KD an Aliquot part of CD, than E doth contain a like Aliquot part of F; which would be contrary to the supposition.

6. There will be a greater Reason of the first quantity to the second, than of the third to the fourth: if the first contains more times any Aliquot part of the second, than the third doth contain a like Aliquot part of the fourth. As 101 hath greater Reason to 10 than 200 to 20; because that 101 contains One Hundred and one times the Tenth part of 10, and 200 contains only One Hundred times the Tenth part of 20. which is 2.

7. The Magnitudes or quantities which are in the same Reason, are called Proportionals.

8. A Proportion or Analogie, is a Similitude of Reason or habitude.

9. A Proportion ought to have at least three terms. For to the end there be similitude of Reason, there must be two Reasons: Now each Reason having two terms, the antecedent and the consequent, it seemeth there ought to be four, as when we say that there is the same Reason of A to B, as of C to D: but because the consequent of the first Reason may be taken for antecedent in the second, three terms

terms may suffice, as when I say that there is the same Reason of A to B, as of B to C.

10. Magnitudes are in continued Proportion, when the Terms between them are taken twice; that is to say, as antecedent and as consequent. As if there be the same Reason of A to B, as of B to C, and of C to D.

11. Then A to C shall be in duplicate Ratio of A to B: and the Ratio of A to D shall be in triplicate Ratio to that of A to B.

It is to be taken notice, that there is a great deal of difference between double Ratio and duplicate Ratio. We say that the Ratio of four to two is double, that is to say, four is the double of two, whence it followeth that the number two, is that which giveth the Name to this Ratio, or rather to the Antecedent of this Ratio. So we say, double, triple, quadruple, quintuple, which are Denominations taken from those numbers, duo, tres, quatuor, quinque, compared with unity: for we better conceive a Reason when its terms are small. But as I have already taken notice, those Denominations fall rather on the Antecedent than on the Reason it self: we call that double, triple, reason or Ratio, when the Antecedent is double or triple to the consequent: but when we say the reason is
dupli-

duplicate, we mean a Reason compounded of two like Reasons, as if there be the same Reason of two to four, as of four to eight; the Reason of two and eight being compounded of the Reason of two and four, and of that of four and eight, which are alike; and as equal the Reason or Ratio of two to eight, shall be duplicated by each. Three to twenty seven, is a duplicated Reason of that of three to nine. The Reason of two to four is called subduple; that is to say, two is the half of four: but the reason of two to eight is duple of the sub-duple; that is to say, that two is the half of the half of eight: as three is the third of the third of twenty seven; where you see there is taken twice the Denominator $\frac{1}{2}$ and $\frac{1}{3}$: In like manner, eight to two is a duplicate reason of eight to four, because eight is double to four, but eight is the double of the double of two. If there be 4 terms in the same continued Reason, that of the first and last is triple to that of the first and second, as if one put these four Numbers, two, four, eight, sixteen; the reason of two to sixteen is triple of two to four; for two is the half of the half of the half of sixteen. As the reason of sixteen to two, is triple of sixteen to eight; for sixteen is the double of eight, and it is the double of the double of the double of two.

12. Magnitudes are homologous, the Antecedents to the Antecedents, and the Consequents to the Consequents. As if there be the same Reason of A to B , as of C to D . A and C are homologous, or Magnitudes of a like Ratio.

The following Definitions are ways of arguing by Proportion, and it is principally to demonstrate the same that this Book is composed.

13. Alternate Reason, or by Permutation, or Exchange, is, when we compare the Antecedents one with the other, as also the consequents. For example, if because there is the same reason of A to B as of C to D , I conclude there is the same reason of A to C as of B to D ; this way of reasoning cannot take place but when the four terms are of the same Species; that is to say, either all four Lines, or Superficies, or Solids. Proposition 16.

14. Converse, or Inverse Reason, is a comparison of the Consequents to the Antecedents. As, if because there is the same reason of A to B as of C to D ; I conclude, there is the same reason of B to A as there is of D to C . Proposition 16.

15. Composition of Reason is a comparison of the Antecedent and Consequent taken together, to the Consequent alone.

alone. *As, if there be the same Reason of A to B as of C to D; I conclude also, that there is the same reason of AB to B as of CD to D. Prop. 18.*

16. Division of Reason is a comparison of the excess of the Antecedent above the Consequent, to the same Consequent. *As, if there be the same reason of AB to B, as of CD to D; I conclude that there is the same reason of A to B, as of CD. Prop. 17.*

17. Conversion of Reason is the comparison of the Antecedent to the difference of the Terms. *As, if there be the same reason of AB to B, as of CD to D; I conclude, that there is the same reason of AB to A, as of CD to C. Proposition 18.*

18. Proportion of Equality is a comparison of the extream Quantities in leaving out those in the middle. *As if there were the same reason of A to B as of E to F, and of B to C as of F to G, and of C to D as of G to H. I draw this Consequence, that there is therefore the same reason of A to D as of E to H.*

19. Proportion of Equality well ranked, is that in which one compareth the Terms, in the same manner of Order; *as in the preceding Example. Prop. 22.*

20. Pro-

20. Proportion of Equality ill ranked, is that in which one compareth the Terms with a different Order. As if there were the same reason of A to B as of G to H, and of B to C as of F to G, and of C to D as of E to F. I draw this Conclusion, that there is the same reason of A to D as of E to H. Prop. 28.

Here is all the ways of arguing by Proportion. There is the same reason of A to B as of C to D; therefore by alternate reason there is the same reason of A to C as of B to D: and by inversed reason, there is the same Reason of B to A as of D to C; and by composition, there is the same reason of AB to B as of CD to D.

By Division of Reason, if there be the same reason of AB to B as of CD to D, there is the same reason of A to B as of C to D; and by Conversion, there is the same reason of AB to A as of CD to C.

By reason of Equality well ranked, if there be the same reason of A to B as of C to D; and also the same reason of B to E as of D to F; there will be the same reason of A to E as of C to F.

By reason of Equality ill ranked, if there be the same reason of A to B as of D to F, and also the same reason of B to E as of C to D; there will be the same reason of A to E as of C to F.

This

The Fifth Book of

This Book contains 25 Propositions of Euclid, to which there has been added 10, which are received. The first 6 of this Book are useful only to prove the following Propositions by the method of Equimultiples; and seeing I do not make use of that method, I begin at the 7th, without changing the Order or Number of the Propositions.

The Demands or Propositions.

Three Magnitudes ABC being proposed, it is required, that it be agreed to, or granted, that there is a 4th Magnitude in possibility, to which the Magnitude C hath the same Reason as the Magnitude A hath to the Magnitude B.

PROPOSITION VII.

THEOREM.

The Magnitudes which are equal have the same reason to a third Magnitude; and one and the same Magnitude hath the same reason to equal Magnitudes.

A,	8.
C,	4.
B,	8.

If the Magnitude A and B are equal, they shall have the same reason to a third Magnitude C.

Demonstration. If one of the two, for example A, should have a greater Reason to the Magnitude C, than B;

A would contain more times a certain aliquot part of C, than B would contain: thence A would be greater than B, contrary to what we have supposed.

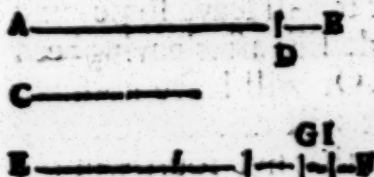
Secondly, I say if A and B are equal, the Magnitude C shall have the same Reason to the Magnitude A, as it hath to the Magnitude B.

Demonstration. If the Magnitude C should have a greater Reason to the Magnitude A, than it hath to the Magnitude B; it ought to contain more times an aliquot of the Magnitude A, than it doth contain alike aliquot part of the Magnitude B. So that that part of A ought to be less than a like aliquot part of B. Thence A should be less than B: which is contrary to the supposition.

PROPOSITION VIII

THEOREM.

THe greatest of two Magnitudes hath a greater Reason to the same third, than the lesser; and the same third hath a lesser Reason to the greater Magnitude than it hath to the lesser.



I suppose that one should compare the Magnitudes A B and C, with the same third EF;

EF; and that AB be greater than C. I say that there is a greater Reason of AB to EF, than of C to EF. Let AD be taken equal to C; and let EF be divided in the middle, and the half again in the middle, and so continuedly until such time one find GF, an aliquot part of EF, to be less than DB.

Demonstration. AD, and C are equal; thence there is the same reason of AD to EF, as there is of C to EF (*by the 7th*) and (*by the 4th definition*) AD shall contain as many times GF an aliquot part of EF, as C containeth the same. Now AB containeth the same once more, seeing DB is greater than GF; whence (*by the 5th definition*) the reason of AB to EF, is greater than that of C to the same third EF.

Secondly I say, that EF hath a lesser reason to AB, than to the Magnitude C. Let there be taken any aliquot part of C, for Example the one fourth, as many times as it can be taken in EF. Let us suppose that it be found therein five times, either it will leave something of the Magnitude EF; after having been taken five times. Or will leave nothing, that is to say, it will measure exactly EF, it is evident that five fourths of the Magnitude AB will be a greater Line, than

than the fourth of C taken five times; so that it cannot be found to be five times in EF. And if the fourth of C taken five times, falleth short as in G: either the fourth of AB taken five times, will reach to F; or fall short in I. If it reacheth to F; there will be the same reason of EF to AB, as of EG to C: by the preceeding Argument, EF to C, hath a greater Reason to C, than to AB. That if the Quarter of AB taken five times reacheth to I; there will the same reason of EI to AB, as of EG and C. Now EI, or EF hath a greater reason than EG to C. Therefore EF to C hath a greater reason than the same EF to AB.

PROPOSITION IX.

THEOREM.

Magnitudes are equal when they have the same reason to a third Magnitude.

[A, B, C] If the Magnitudes A and B have the same Reason to a third Magnitude C: I say A and B are equal.

[12.12.16.] Demonstration. If one of the two: for Example A, was greater than B; it would have a greater reason to the Magnitude C (by the 8th) which would be contrary to the Hypothesis. Se-

Secondly, if the Magnitude C hath the same Reason to the Magnitude A, as it hath to the Magnitude B: I say that A and B are equal, for if A was greater than B, C would have a greater Reason to the Magnitude B, than to the Magnitude A; which would be also contrary to the supposition.

PROPOSITION X.

THEOREM.

The Magnitude which hath the greatest Reason to the same Magnitude, is the greatest, and that is the least to which the same hath the greatest Reason.

[A, B, C.] If there be a greater Reason between A and C, than between B and C: I say that A is greater than B, for if A and B were equal, they would have the same Reason to C: if A were less than B, there would be a greater Reason between B and C, than between A and C. The one and the other is contrary to the Hypothesis.

Secondly, if there be a lesser Reason between C and A, than between C and B. I say that A shall be greater than B. For if A and B were equal, C would have the same Reason to both (*by the 8th.*) If A was less than B, C would have less Reason to B than to A, the one and the other is contrary to what we have supposed.

PRO.

PROPOSITION XI.
THEOREM.

THe Reasons which are equal to the same Third, are also the same one to another.

A, B, C, D, E, F. If there be the same Reason between A and B, as between C and D, and if there be also the same Reason between C and D, as between E and F: I say there will be the same Reason between A and B; as between E and F.

Demonstration. Seeing there is the same Reason between A and B, as there is between C and D; A shall contain as many times any aliquot part whatever of B as C containeth alike aliquot part of D, (*by the 5th Definition*) and in like manner, as many times as C containeth that aliquot part of D; E contains alike aliquot part of F. So then, as many times as A containeth any aliquot part whatever of B; E shall contain also a like aliquot part of F. Therefore there will be the same Reason between A and B; as there is between E and F.

PRO.

PROPOSITION XII THEOREM.

IF several Magnitudes are Proportion-
als, there shall be the same Reason
of one Antecedent to its consequent, as
of all the Antecedent taken together, to
their consequents.

A	B,
3	12
C	D
2.	8.

If there be the same reason
of A to B, as there is of C to
D: I say that there is the
same reason of A and C, taken
together to B and D, as of A
to B.

Demonstration. Seeing there is the
same reason of A to B, as of C to D;
the Magnitude A shall contain as many
times any aliquot part whatever of B,
as C contains a like aliquot part of D;
for Example, the fourth (*by the 5th defi-
nition.*) Now the fourth of B, and the
fourth of D, are one fourth of BD :
so then AC shall contain as many times
one fourth of BD, as A contains the
fourth of B: and what I say of the
fourth verifieth it self of all other ali-
quot parts. There is therefore the same
reason of A to B, as of AC to BD.

PRO.

PROPOSITION XIII.

THEOREM.

IF two equal Reasons the one is greater than a third, the other shall be so likewise.

[AB:CD:EF.] If there be the same reason of A to B, as of C to D, and that there is a greater reason of A to B, than of E to F; I say that there shall be a greater reason of C to D, than of E to F.

Demonstration. Seeing there is a greater reason of A to B, than of E to F; A shall contain more times any aliquot part of B, than E contains a like aliquot part of F, (by the 6th definition.) Now C contains a like aliquot part of D, as many times as A contains that of B, seeing there is the same reason of A to B, as of C to D; so then C contains an aliquot part of D, more times than E containeth a like aliquot part of F; thence there is a greater reason of C to D, than of E to F.

PROPOSITION XIV.

THEOREM.

IF there be the same reason of the first Magnitude to the second, as of the third to the fourth: if the first be greater, equal, or less than the third, the second shall

shall be be greater, equal, or less than the fourth.

[A, B, C, D] If there be the same reason of A to B, as of C to D, I say in the first place, that if A be greater than C, B shall also be greater than D.

Demonstration. Seeing that A is greater than C, there will be (by the 9th) a greater Reason of A to B, than of C to B. Now as A is to B, so is C to D; therefore there shall be a greater reason of C to D, than of C to B; and by consequence (according to the tenth) B shall be greater than D.

I say in the second place, that if A be equal to C, B shall be also equal to D.

Demonstration. Seeing that A and C are equal, there will be the same reason of A to B, as of C to B (by the 7th.) Now as A is to B, so is C to D; thence there is the same reason of C to B, as of C to D; and by consequence, B and D are equal (by the ninth) and in the third place if A be less than C, B shall be also less than D.

Demonstration. Seeing that A is less than C, there will be a lesser reason of A to B, than of C to B, (by the 8th.) Now as A is to B, so is C to D; thence there will be a lesser reason of C to D, than

than of C to B; and by consequence
(according to the 11th.) B shall be less
than D.

PROPOSITION XV.
THEOREM.

The Equimultiples and like aliquot parts, are in the same Reason.

A,	B,	C,	D.
2.	3.	6.	9.
E,	2.	H,	3.
F,	2.	I,	3.
G,	2.	K,	3.

If the Magnitudes C and D are Equimultiples of A and B, their aliquot parts, there shall be the same reason of A to B, as of

C to D; let the Magnitude C be divided into parts equal to A, which shall be E, F, G: Let the Magnitude D be divided into parts equal to B, seeing that C and D are Equimultiples of A and B: there will be as many parts in the one as in the other.

Demonstration. There is the same reason of E to H, of F to I, of G to K, as of A to B; seeing they are equal, thence (by the 12th) there shall be the same reason of E, F, G, to HIK; that is to say, of C to D, as of A to B.

Coroll. The same number of aliquot parts of two Magnitudes, are in the same reason with those Magnitudes:

K for

for seeing there is the same reason of E to H, as of C to D, and of F to I, there will be the same reason of EF to HI, as of C to D.

PROPOSITION XVI THEOREM.

Alternate Reason.

I*F four Magnitudes of the same Species are proportional, they shall also be proportional alternately.*

[A, B, C, D.] If there be the same reason of A to B, as of C to D, and if the four Magnitudes be of the same Species; that is to say, if all four be Lines, or all four Superficies, or all four Solids, there will be the same reason of A to C, as of B to D. Now suppose that there is a greater reason of A to C than of B to D.

Demonstration. Seeing one would have it, that there is a greater reason of A to C, than of B to D, the Magnitude A will contain an aliquot part of C, for Example the third, more times than B contains the third of D. Let A contain the third of C four times, and B the third of D only three times; having divided A into four parts, the
third

third of C shall be once in each: having also divided B into four parts, the third of D shall not be in each. Thence the three fourths of A shall contain the three thirds of C, that is to say, the Magnitude C; and the three fourths of B will not contain the three thirds of D; that is to say, the Magnitude D. Again, seeing there is the same reason of A to B, as of C to D, there will be also the same reason of the three fourths of A to the three fourths of B, as of C to D, (by the Coroll. of the 15th) and (by the 14th) if the three fourths of A be greater than C, the three fourths of B shall be greater than D; although we have demonstrated the contrary.

LEMMA.

IF there be the same reason of the first Magnitude to the second, as of the third to the fourth; an aliquot part of the first shall have the same reason to the second as a like aliquot part of the third hath to the fourth.

16.	3.	32.	6.
A,	B,	C,	D.
E,		F,	
4.		8.	

If there be the same reason of A to B as of C to D, and that E be an aliquot part of A, and F a like aliquot part

The Fifth Book of

part of C; I say there is the same reason of E to B, as of F to D.

Demonstration. If there were a greater reason of E to B than of F to D, E would contain an aliquot part of B more times than F contains a like an aliquot part of D. Then E taken twice, thrice, or four times, would contain aliquot part of B, more times than F taken twice, thrice, or four times, would contain an aliquot part of D. Now E taken four times, is equal to A, as F is to C. So A would contain an aliquot part of B more times than C containeth a like aliquot part of D: thence there would be a greater reason of A to B than of C to D; which is contrary to the Supposition.

C O R O L L A R Y.

Which in Euclid is after the fourth Proposition.

Converse Reason.

IF there be the same Reason of the first to the second, as of the third to the fourth; there shall also be the same Reason of the second to the first, as of the fourth to the third,

If

A,	B,	C,	D,
4.	8.	12.	24.
E,		F,	
1.		3.	

If there be the same reason of the Magnitude A to B, as of C to D; there will be the same reason of B to A as of D to C.

Demonstration. If there were a greater reason of B to A than of D to C, B would contain an aliquot part of A, for example the fourth E, more times than D, contains F the fourth of C. Let us suppose that B should contain eight times the Magnitude E, and D should only contain seven times the magnitude F, and seeing there is the same reason of A to B, as of C to D, there will be also the same reason of E to B, as of F to D, (*by the foregoing Lemma*) and (*by the 15th.*) E taken eight times shall have the same reason to B, as F taken eight times to D. Now E taken eight times is contained in B: thence F taken eight times shall be contained in D; although we have demonstrated the contrary. There is not then a greater reason of B to A, than of D to C.

K 3

PRO.

PROPOSITION XVII.

THEOREM.

Division of Reason.

IF Magnitudes compounded be proportional, they shall be proportional also when divided.

A.	B.	C.	D.	If there be the same reason of AB to B, as of CD to D, there shall be also the same reason of A to B, as of C to D.
5.	3.	10.	6.	

Demonstration. Seeing it is supposed that there is the same reason of AB to B, as of CD to D, AB will contain an aliquot part of B, as many times as CD contains a like aliquot part of D: now this aliquot part is found as many times in B, as the like is found in D: thence taking away B from AB, and D from CD, A will yet have as many aliquot parts of B as C containeth like aliquot parts of D; and by consequence there shall be the same reason of A to B as of C to D.

PRO.

PROPOSITION XVIII. THEOREM.

Composition of Reason.

IF Magnitudes divided be proportional, the same also being compounded shall be proportional.

A,	B,	C	D.
5.	3.	10.	6.

If there be the same reason of A to B as of C to D, there shall be also the same reason of AB to B as of CD to D.

Demonstration. Seeing it is supposed that there is the same reason of A to B as of C to D, A shall contain an aliquot part whatever of B, as many times as C contains a like aliquot part of D. Now the Magnitude B contains any aliquot part of it self as many times as D contains alike of it self: thence adding B to A, and D to C, AB shall contain an aliquot part of B, as many times as CD contain a like aliquot part of D: there is therefore (by the fifth Definition) the same reason of AB to B, as of CD to D.

COROLLARY.

Conversion of Reason.

IF there be the same reason of AB to B, as of CD to D, there shall also be the same reason of AB to A, as of CD to C: for (*by the foregoing*) there will be the same reason of A to B as of C to D: and (*by the Coroll. of the 16th*) there will be the same reason of B to A, as of D to C. and by Composition there will be the same reason of AB to A, as of CD to C.

USE.

WE make use of this way of Arguing in almost all the Parts of the Mathematicks.

PROPOSITION XIX.
THEOREM.

IF the Wholes are in the same reason as the Parts taken away, those which remain shall be in the same reason.

A,	C,
12.	6.
B,	D,
4.	2.

If there be the same reason of the Magnitude AB to the Magnitude CD, as of the part B to the part D; I demonstrate, that there shall be the same reason of A to C as of AB to CD.

De-

Demonstration. It is supposed that there is the same reason of AB to CD, as of B to D: thence, by Exchange (according to the 16th) there shall be the same reason of AB to B, as of CD to D; and by Conversion of reason, there shall be the same reason of AB to A, as of CD to C: and again, by Exchange there shall be the same reason of AB to CD, as of A to C.

U S E.

IN the Rule of Fellowship this Proposition is sometimes made use of; for they do not make use of the Rule of Three for every Associate, but do content themselves for to give to the last the Remainder of the Gain; supposing, that if there be the same Reason of the whole Sum of the Stock to the whole profit, as of the Stock of one Associate, to his part of the Gain; there will be the same reason of the Stock remaining, to the Remainder of the Gain.

The 20th. and 21st. Propositions are not necessary.

PROPOSITION XXII.

THEOREM.

The reason of Equality with Order.

IF one should propose certain terms to which one should compare a like number.

K 5.

ber.

ber in such sort, that those which correspond in the same Rank be proportionals: the first and last shall be proportional.

If the Magnitudes
 $\begin{array}{|c|c|c|c|c|c|} \hline 1. 2. & 6. & 2. & 6. & 3. & 1. \\ \hline A, & B, & C, & D, & E, & F. \\ \hline \end{array}$ A, B, C, and the
 Magnitudes D, E, F,
 are proportional, that is to say, that
 there be the same Reason of A to B, as
 of D to E; of B to C, as of E to F:
 there will be also the same Reason of A
 to C, as of D to F.

Demonstration. If there were a greater Reason of A to C, than of D to F; A would contain an aliquot part of C, for example the half, more times than D would contain the half of F: Let us suppose that the half of C is twelve times in A, and that the half of F is only eleven times in D. Now because there is the same Reason of B to C, as of E to F; the Magnitude B will contain the half of C, as many times as E contains the half of F: Let us suppose that those halves are found six times in B and E. A which contains twelve times the half of C, will have a greater Reason to B, which contains six times the half of C than D, which contains only eleven times the half of, F, to E, which containeth the same six times; there shall be therefore a greater Reason of A to B,

B, than of D to E, although we had supposed the contrary.

PROPOSITION XXIII. THEOREM.

THe reason of equality without order, if two Ranks of terms are in the same reason ill ranked, the first and the last of the one and of the other shall be proportional.

A, B, C, D, E, F, G.
12, 6, 3, 8, 4, 2, 1.

If the Magnitudes A, B, C, and the others D, E, F, in like number be in the same Reason ill ranked, that is to say, that there be the same Reason of A to B, as of E to F; and the same reason of B to C, as of D to E: there will be the same reason of A to C, as of D to F. Let there be the same reason of B to C, as of F to G.

Demonstration. Seeing there is the same reason of A to B, as of E to F; and of B to C, as of F to G; there shall be also the same reason of A to C, as of E to G (*by the 22d.*) Moreover, seeing there is the same reason of B to C as of D to E, and of F to G; there shall be (*by the 11th*) the same reason of D to F, as of E to G: Now as E is to G, so is A to C, as we have already

already proved; thence as A is to C, so is D to F.

PROPOSITION XXIV.
THEOREM.

IF there be the same Reason of the first Magnitude to the second, as of the third to the fourth; and the same of the fifth to the second, as of the sixth to the fourth; there will be the same reason of the first, with the fifth to the second, of the third, with the sixth to the fourth.

E,	F,
4.	6.
6.	2.
9.	3.
A, B,	C, D.

If there be the same Reason of A to B as of C to D, of E to B as of F to D; there shall be the same Reason of AE to B, as of CF to D.

Demonstration. Seeing there is the same reason of A to B as of C to D. A will contain any aliquot part of B whatever, as many times as C contains a like aliquot part of D (*by the 5th Def.*) In like manner E will contain the same aliquot part of B, as many times as F will contain a like aliquot part of D: So then AE will contain any aliquot part of B whatever, as many times as C and F will contain a like aliquot part of D. There shall thence be

be the same reason of AE to B, as of CF to D.

PROPOSITION XXV.

THEOREM.

If four Magnitudes are proportional, the greatest and the least will be greater than the other two.

B.D.
8 6
4 3 4 3
A. C. E. F.

If the four Magnitudes AB, CD, E, F are proportional; let AB be the greatest, and F the least: AB and F shall be greater than CD and E.

Seeing there is the same reason of AB to CD, as of E to F; and seeing it is supposed that AB is greater than E; CD shall be greater than F (*by the 14th*) take from AB the Magnitude A equal to E; and from CD the Magnitude C equal to F.

Demonstration. Seeing there is the same reason of AB to CD, as of A to C; there will be also the same reason of B to D, as of AB to CD (*by the 19th*) and AB being supposed greater than CD; B will be greater than D. Now if A and E be added which are equal, E and F which are also equal; A and F shall be equal to C and E. And adding to the first, B which is greatest; and

and to the second D which is lesser;
AB and F shall be greater than CD,
and E.

USE.

IN this Proposition is demonstrated one
Propriety of Geometrical proportion-
ality which serveth thereto as a differ-
ence which distinguisheth it from Arith-
metical proportionality; for in this last,
the two middle Terms are equal to the
two extremes; and in the Geometrical
the greater and lesser are greater.

Then the other two, although the nine
following Propositions are not of Euclid,
although I ought not to omit them, because
several do make use of them, and do cite
them as if they were thereof.

PROPOSITION XXVI.
THEOREM.

IF there be a greater Reason of the
first to the second, than of the third
to the fourth; the fourth will have a
greater Reason to the third, than the se-
cond to the first.

9.	4.	6.	3.
A,	B,	C,	D,
E,			
8.			

If there be a greater
reason of A to B, than
of C to D; there shall
be a greater reason of
D to C, than of B to A

Let

Let us suppose there were the same reason of E to B, as of C to D: A shall be greater than E, (by the 10th.)

Demonstration. There is the same reason of E to B, as of C to D; thence (by the Coroll. of the 10th) there shall be the same reason of D to C, as of B to E. Now B hath a greater reason to E, than to A, (by the 8th) thence there is a greater reason of D to C, than of B to A.

PROPOSITION XXVII.

THEOREM.

If there be a greater Reason of the first to the second, than of the third to the fourth, there shall be also a greater Reason of the first to the third, than of the second to the fourth.

9.	4.	6.	3.
A,	B,	C,	D.
E,			
8.			

If there be a greater reason of A to B, than of C to D; I demonstrate that there shall be a greater rea-

son of A to C, than of B to D. Let there be the same reason of E to B, as of C to D; A shall be greater than E.

Demonstration. There is the same reason of E to B, as of C to D; thence (by the 16th) there shall be the same reason of E to C, as of B to D. And be-

because A is greater than E, the reason of A to C shall be greater than that of E to C. There is therefore a greater reason of A to C, than of B to D.

PROPOSITION XXVIII. THEOREM.

If there be a greater reason of the first to the second, than of the third to the fourth; there shall be also a greater reason of the first and second to the second, than of the third and fourth to the fourth.

9.	4.	6.	3.
A,	B,	C,	D,
E,			
B.	.		

If there be a greater reason of A to B, than of C to D; there shall be also a greater reason of AB to B, than of CD to D. Let us suppose there is the same reason of E to B, as of C to D.

Demonstration. There is the same reason of E to B, as of C to D: Thence (*by the 18th*) there shall be the same reason of EB to B, as of CD to D: and AB being greater than EB, there shall be a greater reason of AB to B, than of EB to B; and by consequence than of CD to D.

PRO-

PROPOSITION XXIX. THEOREM.

IF the first and second have a greater reason to the second than the third and fourth to the fourth; the first to the second shall have a greater reason than the third to the fourth.

9.	4.	6.	3.
A,	B,	C,	D,
E,			
8.			

If there be a greater reason of AB to B, than of CD to D; there shall be also greater reason of A to B, than of C to

D. Let us suppose that the reason of EB to B is the same with that of CD to D; EB shall be less than AB, and E less than A.

Demonstration. It is supposed that EB is to B in the same reason as CD is to D: thence in dividing (by the 17th) there shall be the same Reason of E to B, as of C to D: And A being greater than B, the Reason of A to B shall be greater than that of C to D.

PROPOSITION XXX. THEOREM.

IF the first and second have a greater reason to the second, than the third and fourth to the fourth: The first and second shall have a lesser reason to the

the first, than the third and fourth to the third.

If AB have a greater Reason to B than CD to D : AB shall have a lesser reason to A , than CD to C .

Demonstration. We suppose that the reason of AB to B , is greater than that of CD to D ; there shall be a greater reason of A to B , than of C to D , (by the 29th) and (by the 26th) there shall be a greater reason of D to C , than of B to A . Thence by composition (by the 28th) the reason of CD to C shall be greater than of AB to A .

PROPOSITION XXXI. THEOREM.

If several Magnitudes are in a greater reason than a like number of other Magnitudes ranked in the same order; the first of the first rank shall have a greater reason to the last, than the first of the second rank to the last.

If there be a greater reason of A to B , than of D to E ; and if B have a greater reason to C , than E to F ; there shall be a greater Reason of A to C , than of D to F .

Demon-

PROPOSITION XXXII.
THEOREM.

B,	F,
12.	3.
13.	6.
A,	C,
E,	H, I, K,

B.	F.			
12.	3.			
13.	6.	2.	4.	2. 1.
A, C.	E.	H.	I, K.	

be the same reason of C to F, as of H to I; F shall be greater than E.

Demonstration. Seeing we suppose that there is the same reason of B to C, as of I to K; and of C to F, as of H to I: there will be also the same reason of B to F, as of H to K (*by the 23d.*) Now there is a greater reason of A to F than of B to F (*by the 8th.*) and the Reason of A to E is greater than that of A to F; seeing F is greater than E, there shall thence be a greater reason of A to E, than of H to K.

PROPOSITION XXXIII. THEOREM.

I*f the whole hath a greater Reason to the whole, than a part to a part; the Remainder shall have a greater reason to the Remainder, than the whole to the whole.*

13. 4. 6. 2.
A, B, C, D. If there be a greater Reason of AB to CD, than of B to D; there shall be a greater Reason of A to C, than of AB to CD.

Demonstration: We suppose that there is a greater Reason of AB to CD, than of B to D: thence (*by the 26th*) there will be a greater Reason of AB, to B, than of CD to D: and (*by the 32d*)

32d) there will be a lesser Reason of AB to CD, than of A to C.

PROPOSITION XXXIV. THEOREM.

IF there be proposed Two ranks of Magnitudes; and if the Reason of the first of the first, to the first of the second, be greater than that of the second, to the second; and this greater than that of the Third to the third: there shall be a greater Reason of all the first ranks, to all the second, than of all the first rank, excepting the first, to all the second rank excepting the first, but it shall be less than the first of the first rank to the first of the second: and in fine greater than the last of the first, to the last of the second.

12.	6.	4.	4.	3.	2.
A,	B,	C	E,	F,	G,

If there be a greater Reason of A to E, than of B to F; and if the Reason of B to F be greater than that of C to G: I say A, B, C, have a greater Reason to E, F, G, than C to G.

Demonstration. There is a greater Reason of A to E, than of B to F; there shall be also a greater Reason of A to B, than of E to F; and by Composition the Reason of AB to B shall be greater than

than of EF to F; and by exchange, there shall be a greater Reason of AB to EF, than of B to F. Now, the Reason of B to F is greater than that of C to G. Thence the Reason of AB to EF, is greater than that of C to G: and by composition, there shall be a greater Reason of A, B, C, to E, F, G, than of C to G.

I say in the second place, that the Reason of A, B, C, to E, F, G, is greater than the Reason of BC to FG.

Demonstration. It is supposed that there is greater Reason of A to E, than of B to F: and by exchange, the Reason of A to B, is greater than that of E to F: and by composition, there shall be a greater Reason of AB to B, than of EF to F: and by exchange, the Reason of AB to EF shall be greater than that of B to F. Moreover, seeing there is a greater Reason of the whole AB, to EF, than of the part B to F: A shall have a greater Reason to E, than AB to EF; and there shall be a greater Reason of B to F, than of BC to FG. And by exchange, there shall be a greater Reason of A to BC, than of E to FG; and by composition, there shall be a greater Reason of ABC, to EFG, than of BC to FG.

I say in the third place that there is a greater Reason of A to B, than of A, B, C, to E, F, G.

Demonstration. We have already demonstrated that there is a greater Reason of A, B, C, to E, F, G, than of the part BC to the part FG: There shall thence be a greater Reason of A to E, than of A, B, C, to E, F, G, (*by the 32d.*)

The end of the Fifth Book.

THE
SIXTH BOOK
OF
Euclid's Elements.

THis Book explaineth and beginneth to apply particular matters of the Doctrine of Proportions, which the preceding Book Explaineth but in General. It beginneth with the most easiest figures, that is to say, Triangles, giving

giving Rules to determine not only the Proportion of their Sides, but also that of their capacity, area or superficies. Then it teacheth to find Proportional Lines, and to augment or diminish any figure whatever, according to a given Ratio. It Demonstrateth the Rule of Three; it applieth the forty seventh of the first, to all sorts of Figures. In fine, it giveth ~~us~~ the most easie and most certain principles to conduct us in all sorts of Measuring.

THE DEFINITIONS.

Def. I.
Fig. I.
Plate 6.

1. **R**ight Lined Figures are like, when that they have all their Angles equal, and the sides which formeth those Angles Proportional.

Fig. I.

As the Triangles ABC , DEF , shall be like, if the Angles A and D ; B and E ; C and F are equal; and if there be the same Ratio of AB to AC , as of DE to DF ; and of AB to CB , as of DE to EF .

Fig. II.

2. Figures are reciprocal, when they may be compared after such sort, that the Antecedent of one Reason, and the Consequent of the other, be found in the same Figure.

That is, when the Analogie beginneth in one figure, and endeth in the same. As if there be the same Reason of AB to CD ; as of DE to EF .

3. A

3. A Line is divided into extreme and mean Proportion or Reason; when there is the same reason of the whole Line to its greatest part, as of its greatest part, to its lesser part.

As if there be the same reason of AB to AC, as of AC to CB; the Line AB shall be divided in the point C in extreme and mean proportion.

4. The Altitude or height of a figure, is the length of the Perpendicular drawn from the top thereof to its Base.

As in the Triangles ABC, EFG, the Perpendiculars EH, AD, whether it fall without or within the Triangles, is their height, or Altitude. Triangles and Parallelograms, whose heights are equal, may be placed between the same Parallels, for having placed their Bases on the same Line HC; if the Perpendiculars DA, HE, are equal, the Lines EA, HC, shall be parallel.

5. A Reason is composed of several Reasons, when the Quantities of those Reasons being multiplied, make a third Reason.

It is to be taken notice of, that a reason, at least the rational, hath its name taken from some number which specifieth the reason or habitude of the Antecedent of that Reason, to its consequent.

L

As

As when there is proposed two Magnitudes or Quantities, the one of twelve Foot, and the other of six, we say that the reason of twelve to six is double. In like manner, when there is proposed two Magnitudes, four and twelve; we would say it is a Subtriple Reason; and $\frac{1}{3}$ is the Denominator thereof, which signifieth that there is the same reason of four to twelve, as of $\frac{1}{3}$ to one, or as of one to three. One may call this Denominator, the Quantity of the reason. Let there be proposed three terms, twelve, six, two: The first reason of twelve to six is double, its Denominator is two; the reason of six to two is triple, its Denominator is three: the reason of twelve to two is compounded of the reason of twelve to six, and of that of six to two; to have the Denominator or the reason of twelve to two, which is compounded of double, and of triple, multiply three by two, and you shall have six; thence the reason of twelve to two is Sextuple. This is that which Mathematicians understand by composition of Reason, altho' it ought to be called Multiplication of Reason.

PRO.

PROPOSITION I. THEOREM.

Parallelograms and Triangles of equal height have the same reason as their Bases.

Let there be proposed the Triangles ACG , DEM , of equal height, in such sort that they may be placed between the same Parallels AD , GM : I say, that there shall be the same reason of the Base GC to the Base EM , as of the Triangle ACG , to the Triangle DEM . Let the Base EM be divided into as many equal parts as you please, and let these be drawn to each Division the Lines DF , DH , &c. Let also the Line GC be divided into parts equal to those of EM , and let be drawn Lines to each division from the top A : All those little Triangles which are made in the two great Triangles, are between the same Parallels, and they have equal Bases; they are thence equal (by the 28th.)

Demonstration. The Base GC , contains as many aliquot parts of the Line EM , as could be found parts equal to EF : Now as many times as there are in the Base GC parts equal to EF ; so many times the Triangle ACG contain-

eth the little Triangles equal to those which are in the Triangle DEM; which being equal among themselves, are its aliquot parts: thence as many times as the Base GC containeth aliquot parts of EM, so many times the Triangle AGC containeth aliquot parts of the Triangle DEM; which will happen in all manner of Divisions. There is thence the same reason of the Base GC, to the Base EM, as of the Triangle AGC, to the Triangle DEM.

Coroll. Parallelograms drawn on the same Bases, and that are between the same parallel Lines, are double to the Triangle. (*by the 4th*) they are thence in the same Reason, as Triangles; that is to say, in the same Reason as their Bases.

U S E.

Fig. I.

THis Proposition is not only necessary to demonstrate those which follow, but we may make use thereof in dividing of Land. Let there be proposed a Trapezium AECD, which hath its sides AD, BC, parallel, and admit one would cut off a third part: Let CE be made equal to AD; and EG the third part of BE. Draw AG. I say that the Triangles ABG is the third of the Trapezium ABCD.

De-

Demonstration. The Triangles ADE, FCE, are equiangled, because of the Parallel AD, CE; and they have their Sides AD, CE, equal: They are thence equal (by the 26th of the 1st) and by consequence the Triangle ABE is equal to the Trapezium. Now the Triangle ABG is the third part of the Triangle ABE by the preceding Proposition, thence the Triangle AEG is the third part of the Trapezium ABCD.

PROPOSITION II.

THEOREM.

A Line being drawn in a Triangle Parallel to its Base, divideth its sides proportionately, and if a Line divideth proportionally the sides of a Triangle, it shall be parallel to its Base.

If in the Triangle ABC, the Line DE, is parallel to the Base BC; the Sides AB, AC, shall be divided proportionally; that is to say, that there shall be the same reason of AD to DB, as of AE to EC. Draw the Lines DE, BE. The Triangles DBE, DEC, which have the same Base DE, and are between the same Parallels DE, BC; are equal (by the 37th of the 1st.)

Demonstration. The Triangles ADE, DBE, have the same point E for their

vertical, if we take AD, DB, for their Bases; and if one should draw through the point E, a parallel to AB, they would be both between the same Parallels; they shall have thence the same reason as their Bases (*by the 1st*) that is to say, that there is the same reason of AD to DB, as of the Triangle ADE to the Triangle DBE, or to its equal CED. Now there is the same reason of the Triangle ADE to the Triangle CED, as of the Base AE to EC. There is therefore the same reason of AD to DB, as of AE to EC.

And if there be the same reason of AE to EC, as of AD to DB: I say that the Lines DE, BG, would then be Parallels.

Demonstration. There is the same reason of AD to DB, as of the Triangle ADE to the Triangle DBE (*by the 1st*) there is also the same reason of AE to EC, as of the Triangle ADE to the Triangle DEC: consequently there is the same reason of the Triangle ADE to the Triangle BDE, as of the same Triangle ADE to the Triangle CED. So then (*by the 7th of the 5th*) the Triangles BDE, CED, are equal: And (*by the 39th of the 1st*) they are between the same Parallels.

USE.

U S E.

THis Proposition is absolutely necessary, in the following Propositions, one may make use thereof in measuring as in the following figure: If it were required to measure the height BE; having the length of the Staff DA, there is the same Reason of CD to DA, as of BC to BE.

PROPOSITION III.

THEOREM.

That Line which divideth the Angle of a Triangle, into two equal parts, divideth its Base in two parts, which are in the same reason to each other as are their Sides. And if that Line divideth the Base into parts proportional to the Sides, it shall divide the Angle into two equally.

If the Line AD divideth the Angle BAC into two equal parts; there shall be the same reason of AB to AC, as of BD to DC. Continue the Side CA, and make AE equal to AB; then draw the Line EB.

Demonstration. The exterior Angle CAB is equal to the two interior Angles AEB, ABE; which being equal (by the 5th of the 1st) seeing the sides

The Sixth Book of

AE, AB, are equal; the Angle BAD, the half of BAC, shall be equal to one of them; that is to say to the Angle ABE. Thence (by the 27th of the 1st) the Lines AD, EB, are parallel; and (by the 2d) there is the same reason of EA, or AB to AC, as of BD to DC.

Secondly, If there be the same reason of AB, to AC, as of BD to DC, the Angle BAC shall be divided into two equally.

Demonstration. There is the same reason of AB or AE to AC, as of BD to DC: thence the Lines EB, AD, are parallel; and (by the 29th of the 1st) the Alternate Angles EBA, BAD, the internal BEA, and the external DAC, shall be equal; and the Angles EBA, AEB, being equal; the Angles BAD, DAC, shall be so likewise. Wherefore the Angle BAC hath been divided equally.

U S E.

WE make use of this Proposition to attain to the Proportion of the Sides.

PROQ

PROPOSITION IV. 1 to 28

THEOREM.

Equiangular Triangles have their Sides Proportional.

If the Triangles ABC, DCE, are equiangular; that is to say, that the Angles ABC, DCE, BAC, CDE, be equal: There will be the same Reason of BA to BC, as of CD to CE. In like manner the reason of BA to AC, shall be the same with that of CD to DE. Joyn the Triangles after such a manner, that their Bases BC, CE, be on the same Line; and continue the sides ED, BA: seeing the Angles ACB, DEC, are equal; the Lines AC, EF, are parallel; and so CD, BF, (by the 29th. of the 1st) and AF, DC, shall be a parallelogram.

Demonstration. In the Triangle BFE, AC, is parallel to the Base FE, thence (by the 2d) there shall be the same reason of BA to AF, or CD, as of BC to CE; (and by exchange) there shall be the same reason of AB to BC, as of DC to CE. In like manner in the same Triangle, CD being parallel to the Base BF; there shall be the same Reason of FD, or AC to DE, as of BC to CE (by the 3d) and by exchange, there

shall be the same reason of AC to BC,
as of DE to CE.

USE.

THis Proposition is of a great extent, and may pass for a universal Principle in all sorts of Measuring. For in the first place the ordinary practice in measuring inaccessible Lines, by making a little Triangle, like unto that which is made or imagined to be made on the ground, is founded on this Proposition, as also the greatest part of those Instruments, on which are made Triangles like unto those that we would measure, as the Geometrical Square, Sinical Quadrant, Jacobs Staff, and others. Moreover we could not take the plane of a place, but by this Proposition: wherefore to explain its uses, we should be forced to bring in the first Book of practical Geometry.

PROPOSITION V.

THEOREM.

Triangles whose sides are proportional, are equiangular.

If the Triangles ABC, DEF, have their sides proportional, that is to say, if there be the same reason of AB to BC, as of DE to EF; as also if there be

be the same reason of AB to AC, as of DE to DF, the Angles ABC, DEF, A and D, C and F shall be equal. Make the Angle FEG equal to the Angle B; and EFG equal to the Angle C.

Demonstration. The Triangles ABC, EFG, have two Angles equal; they are thence equiangular (by the Cor. of the 32^d of the 1st) and (by the 4th) there is the same reason of DE to EF, as of EG to EF. Now it is supposed that there is the same reason of DE to EF, as of EG to EF. Thence (by the 7th of the 5th) DE, EG, are equal. In like manner DF, FG, are also equal, and (by the 8th of the 1st) the Triangles DEF, GEF, are equiangular. Now the Angle GEF is equal to the Angle B; and the Angle DFE, to the Angle C. So that the Triangles ABC, DEF, are equiangular.

PROPOSITION VI.

THEOREM.

Triangles which have their sides proportional, which include an equal Angle, are equiangular.

If the Angles B and E of the Triangles ABC, DEF, being equal, there be the same reason of AB to BC, as of DE to EF; the Triangles ABC, DEF, shall be equiangular. Make the Angle FEG,
equal

equal to the Angle B, and the Angle EFG, equal to the Angle C.

Demonstra. The Triangle ABC, EGF, are equiangular (by the Cor. of the 32^d of the 1st) there is thence the same reason of AB, to BC, as of EG to EF, (by the 4th.) Now as AB to BC, so is DE to EF; there is then the same reason of DE to EF, as of GE to EF. So then (by the 7th of the 5th) DE, EG, are equal; and the Triangles DEF, GEF, which have their Angles DEF, GEF, each of them equal to the Angle B, and the Sides DE, EG, equal, with the Side EF common; they shall be equal in every respect (by the 4th of the 1st) they are thence equiangular; and the Triangle EGF being equal to the Triangle ABC, the Triangles ABC, DEF, are equiangular.

The Seventh Proposition is unnecessary.

PROPOSITION VIII. THEOREM.

A Perpendicular being drawn from the Right Angle of a Right Angled Triangle to the opposite side, divideth the same into Two Triangles which are a like thereto.

If from the Right Angle ABC
be

be drawn a perpendicular BD to the opposite side AC, it will divide the Right Angled Triangle ABC into Two Triangles ADB, BDC, which shall be like, or equiangular to the Triangle ABC.

Demonstration. The Triangles ABC, ADB, have the same Angle A; the Angle ADB, ABC, are right: they are thence equiangular (by the Cor. 2. of the 32d. of the 1st.). In like manner the Triangles BDC, ABC, have the Angle C common; and the Angles ABC, BDC, being right, they are also equal. Thence the Triangles ABC, DBC, are like.

U S E.

WE measure inaccessible distances by a Square, according to this Proposition. For example, if we would measure the distance DC, having drawn the perpendicular DB, and having put a Square at the Point B, in such manner, that by looking over one of its Sides BC, I see the Point C, and over its other Side I see the Point A; it is evident that there will be the same reason of AD to DB, as of DB to DC. So that multiplying DB by its self, and dividing that product by AD, the Quotient shall be DC.

PROPOSITION IX.

PROBLEM.

To cut off from a Line any part required.

Let there be proposed the Line AB, from which it is required to cut off three Fifths. Make the Angle BCD at discretion, take in one of those Lines CD, five equal parts; and let CF be three of the same, and CE be equal to AB. Then draw the Line DE; after which draw FG parallel to DE; the Line CG will contain three Fifths of CE or AB.

Demonstration. In the Triangle ECD, FG being parallel to the Base DE, there will be the same reason of CF to FD, as of CG to GE (by the second) and by composition (by the 18th of the 5th.) there shall be the same reason of CG to CE, as of CF to CD. Now CF contains three fifths of CD: wherefore CG shall contain three fifths of CE, or AB.

PROPOSITION X.

PROBLEM.

TO divide a Line after the same manner as another Line is divided.

If one would divide the Line AB after

ter

ter the same manner as the Line AC is divided: Join those Lines, making an Angle at pleasure, as CAB: Draw the Line BC, and the parallels EO, FV, and the Line AB shall be divided after the same manner as AC.

Demonstration. Seeing that in the Triangle BAC, the Line HX hath been drawn parallel to the Base BC; it will divide the Sides AB, AC, proportionally (*by the second*) it is the same with all the other parallels.

To do the same with more facility, one may draw BD parallel to AC, and put off the same Divisions of AC on BD, then draw the Lines from the one to the other.

PROPOSITION XI. THEOREM.

TO find a third Proportional to two given Lines.

It is required to find a third proportional to the Lines AB, BC, that is to say, that there may be the same reason of AB to BC, as of BC to the Line required. Make at discretion the Angle EAC; put off one after the other, the Lines AB, BC; and let AD be equal to BC: draw the Lines BD, and its parallel CE. The Line DE shall be that which you require. De-

Demonstration. In the Triangle EAC, the Line DB is parallel to the Base CE: There is thence (by the 2d) the same reason of AB to BC, as of AD, or BC to DE.

PROPOSITION XII.

PROBLEM.

To find a fourth Proportional to three Lines given.

Let there be proposed three Lines AB, BC, DE, to which must be found a fourth proportional, make an Angle as EAC, at discretion; take on AC the Lines AB, BC, and on AF, the Line AD equal to DE: Then draw DB, and its parallel FC. I say that DF is the Line you seek for; that is to say, that there is the same reason of AB to BC, as of DE, or AD to DF.

Demonstration. In the Triangle FAC, the Line DB is parallel to the Base FC; there is thence the same reason of AB to BC, as of AD to DF (by the 2d.)

U S E.

The use of the Compass of Proportion (or Sector) is established on these Propositions: For we divide a Line as we please, by the Compass of Proportion: we do the Rule of Three without making

making use of Arithmetick: we extract the Square Root and Cube Root: we double the Cube: we measure all sorts of Triangles: we find the Content of Superficies; and the Solidity of Bodies; we augment or diminish any Figure what-ever, according to what Proportion we please; and all those uses are demonstrated by the foregoing Propositions.

PROPOSITION XIII.

PROBLEM

TO find a mean Proportional between two Lines.

If you would have a mean Proportional between the Lines LV, VR: having joyned them together on a strait Line, divide the Line LR into two equal parts in the point M; and having described a Semi-circle LTR on the Center M; draw the perpendicular VT; it shall be a mean Proportional between LV, VR. Draw the Lines LT, TR.

Demonstration. The Angle LTR, described in a Semi-circle, is right (by the 31st of the 3d) and (by the 8th) the Triangles LVT, TVR, are like; there is thence the same reason in the Triangle LVT, of LV to VT, as of VT to VR in the Triangle TVR, (by the 4th.)

Q. E. D.

So

So then VT is a mean Proportional between LV and VR.

USE.

WE reduce to a Square any Rectangular Parallelogram whatever by this Proposition. For Example, in the Rectangle comprehended under LV, VR, I will demonstrate hereafter, that the Square of VT is equal to a Rectangle comprehended under LV, and VR.

PROPOSITION XIV.

THEOREM.

Equiangular and equal Parallelograms have their Sides reciprocal and equiangular Parallelograms whose sides are reciprocal, are equal.

If the Parallelograms L and M be equiangular and equal, they shall have their sides reciprocal; that is to say, that there shall be the same reason of CD to DE, as of FD to DB. For seeing they have their Angles equal, they may be joyned after such manner that their joyned sides CD, DE, be on one straight Line (by the 15th of the 1st) continue the Sides AB, GE; you will have completed the Parallelogram BDEF.

Demonstration. ¶ Seeing the Parallelogram

logram L and M are equal, they shall have the same reason to the Parallelogram BDEH. Now the reason of the Parallelogram L, to the Parallelogram BDEH, is the same with that of the Base CD to the Base DE (*by the 1st*) and that of the Parallelogram M, or DFGE, is the same with that of the Base FD to the Base BD. Thence there is the same reason of CD to DE, as of FD to BD.

Secondly, if the equiangular Parallelograms L and M, have their Sides reciprocal, they shall be equal.

Demonstration. The Sides of the Parallelograms are reciprocal; that is to say, that there is the same Reason of CD to DE, as of FD to BD: Now as the Base CD is to DE, so is the Parallelogram L to the Parallelogram BDEH (*by the 1st*) and as FD is to BD, so is the parallelogram M to BEDH; there is thence the same reason of L to BDEH, as of M to the same BDEH; so then (*by the 7th of the 5th*) the parallelograms L and M are equal.

PROPOSITION XV.

THEOREM.

Equal Triangles which have one Angle equal, have the sides which form that

What Angle reciprocal; and if their sides be reciprocal, they shall be equal.

If the Triangles F and G, being equal, have their Angles ACB, ECD, equal: their sides about that Angle shall be reciprocal; that is to say, that there shall be the same Ratio of BC to CE, as of CD to CA. Dispose the Triangles after such a manner, that the Sides CD, CA, be one straight Line: seeing the Angles ACB, ECD, are supposed equal, the Lines BC, CE, shall be also a straight Line, (*by the 14th of the 1st.*) draw the Line AE.

Demonstration. There is the same Ratio of the Triangle ABC to the Triangle ACE, as of the Triangle ECD, equal to the first, to the same Triangle ACE, (*by the 7th of the 5th.*) Now as ABC, is to ACE, so is the Base BC to the Base CE (*by the 1st*) seeing they have the same vertical A; and as ECD is to ACE, so is the Base CD to CA. Now if it be supposed that the sides are reciprocal; that is to say, that there be the same Ratio of BC to CE, as of CD to CA; the Triangles ABC, CDE, shall be equal, because they would then have the same Ratio to the Triangle ACE.

PRO

PROPOSITION XVI.

THEOREM.

IF four Lines be proportional, the Rectangle comprehended under the first and fourth, is equal to the Rectangle comprehended under the second and third, and if the Rectangle comprehended under the extremes, be equal to the Rectangle comprehended under the means, then are the four Lines proportional.

If the Lines AB, CD , be proportional, that is to say, that there is the same Ratio of A to B , as of C to D , the Rectangle comprehended under the first A , and the fourth D , shall be equal to the Rectangle comprehended under B and C .

Demonstration. The Rectangles have their Angles equal, seeing it is Right; they have also their Sides reciprocal; they are thence equal (by the 16th.)

In like manner, if they be equal, their Sides are reciprocal; that is to say, that there is the same Ratio of A to B , as of C to D .

PROPOSITION XVII.

THEOREM.

IF three Lines be proportional, the Rectangle comprehended under the first

first and third is equal to the Square of the mean, and if the Square of the mean be equal to the Rectangle of the extremes, the three Lines are then proportional.

If the three Lines A, B, D, be proportional; the Rectangle comprehended under A, and under D, shall be equal to the Square of B. Take C equal to B; there shall be the same Ratio of A to B, as of C to D: thence the four Lines A, B, C, D, are proportional.

Demonstration. The Rectangle under A and D shall be equal to the Rectangle under B and C (*by the foregoing:*) Now this last Rectangle is a Square, seeing the Lines B, and C, are equal: thence the Rectangle under A and D is equal to the Square of B.

In like manner, if the Rectangle under A and D be equal to the Square of B, there shall be the same Ratio of A to B, as of C to D; and seeing that B and C are equal, there shall be the same Ratio of A to B, as of B to D.

U S E.

THese Four Propositions demonstrate that Rule of Arithmetick which we commonly call the Rule of Three, and consequently the Rule of Fellowship, false position, and all other which are performed

formed by Proportion. For example, let there be proposed these three Numbers, A Eight, B six, C four, and it is required to find the fourth proportional. Suppose it to be found, and let it be D. The Rectangle comprehended under A and D is equal to the Rectangle comprehended under B and C. Now I can have this Rectangle by Multiplying B by C; that is to say, six by four, and I shall have twenty four, thence the Rectangle comprehended under A and D, is twenty four, wherefore dividing the same by A eight, the Quotient three is the number I look for.

PROPOSITION XVIII.

THEOREM.

TO describe a Polygon like to another, on a Line given.

There is proposed the Line AB, on which one would describe a Polygon like unto the Polygon CFDE. Having divided the Polygon CFDE into Triangles, make on the Line AB a Triangle ABH like unto the Triangle CFE, that is to say, make the Angle ABH equal to the Angle CFE, and BAH equal to FCE. So then the Triangles ABH, CFE, shall be equiangled (by the 32d. of the first.) make also on BH, a Triangle equiangled to FDE.

De-

Demonstration. Seeing the Triangles which are parts of the Poligons, are equiangular, the two Poligons are equiangular. Moreover, seeing the Triangles ABH, CFE, are equiangular, there is the same Ratio of AB to BH, as of CF to FE, (*by the 8th.*) In like manner, the Triangles HBG, EFD, being equiangular, there shall be the same Ratio of BH to BG, as of FE to FD; and by equality there shall be the same Ratio of AB to BG, as of CF to FD. And so of the rest of the sides. Thence (*by the first Definition.*) the Poligons are like to each other.

U S E.

IT is on this Proposition we establish the greatest part of the practical ways to take the plane of a Place, of an Edifice, of a Field, of a Forrest, or of a whole Country; for making use of the equal parts of a Line for Feet or for Chains; we describe a figure like unto the Prototype, but lesser, in which we may see the Proportion of all its Lines. And because it is easier on paper than on the ground; we may comprehend in this Proposition all Geodeses, all Chorography, all Geographical Charts, and ways of reducing of the greater into a lesser; wherefore
this

this Proposition extends almost to all Arts, in which it is necessary to take a Design or Model.

PROPOSITION XIX. THEOREM.

Like Triangles are in duplicate Ratio to their homologous sides.

If the Triangles ABC, DEF, be like or equiangular; they shall be in duplicate Ratio of their homologous Sides BC, EF; that is to say, that the Ratio of the Triangle ABC, to the Triangle DEF shall be in duplicate Ratio of BC to EF; wherefore, by seeking the third Proportional HI to the Lines BC, EF; or so making it, that there may be the same Ratio of BC to EF, as of EF to HI; the Triangle ABC shall have the same Ratio to the Triangle DEF, as the Line BC hath to the Line HI. Which is called a duplicate Ratio, or doubled Reason (*by the 11th. Def. of the fifth.*) Let BG and HI be equal; and let the Line AG be drawn.

Demonstration. The Angle B and E of the Triangles ABG, DEF, are equal: also seeing the Triangle ABC, DEF, are like; there shall be the same Ratio (*by the 4th*) of AB to DE, as of BC to EF: Now as BC is to EF, so is EF to HI,

M

OF

or BG: thence as AB is to DE, so is EF to BG: and consequently the sides of the Triangles ABG, DEF, being reciprocal, the Triangles shall be equal (*by the 15th.*) Now (*by the first,*) the Triangle ABC hath the same Ratio to the Triangle ABG, as BC to BG, or HI: thence the Triangle ABC hath the same Ratio to the Triangle DEF, as BC to HI.

U S E.

THose Propositions do correct the opinions of many, who easily imagine, that like figures have the same Ratio as their Sides. For example, let there be proposed two Squares, Two Pentagons, two Hexagons, two Circles, and let the sides of the first be double to the side of the second; the first figure shall be quadruple to the second. If the side of the first be triple to that of the second, the first figure shall be nine times greater than the second. So that to find a Square triple to another, there must be sought a mean Proportional between one and three, which would be almost $1\frac{1}{2}$ for the side of the triple figure.

OF PROPOSITION XX.

THEOREM.

Like Polygons may be divided into so many like Triangles, and they are in duplicate Ratio to their homologous sides.

If the Polygons ABCDE, GHIML, be like; they may be divided into so many like Triangles, and which shall be like parts of their whole. Draw the Lines AC, AD, GI, GL.

Demonstration. Seeing the Polygons are alike, their Angles B and H shall be equal, and there shall be the same Ratio of AB to BC, as of GH to HI, (by the 15th) thence the Triangles ABC, GHI are alike: and (by the 4th) there shall be the same Ratio of BC to CA, as of HI to GL. Moreover, seeing there is the same Ratio of CD to BC, as of IL to IH, and the same Ratio of BC to CA, as of HI to GL. There shall be by equality the same Ratio of CD to CA, as of IL to GL. Now the Angles BCD, and HIL, being equal, if you take away the equal Angles ACB, GIH, the Angles ACD, GIL, shall be equal. Thence the Triangles ACD, GIL, shall be like, (by the 15th.) So that it is easie

after the same manner to go round about the Angles of the Polygons, and to prove they are similar or like.

I further add, that the Triangles are in the same Ratio as are the Polygons.

Demonstration. Seeing all the Triangles are like, their sides shall be in the same Ratio (*by the 4th.*) Now each Triangle is to its like, in duplicate Ratio of its homologous sides (*by the 19th*) thence each Triangle of one Polygon, to each Triangle of the other Polygon, is in duplicate Ratio to the sides; which being the same; there shall be the same Ratio of each Triangle to its like, as of all the Triangles of one Polygon, to all the Triangles of the other Polygon (*by the 12th of the 5th*) that is to say, as of the one Polygon to the other.

* *Coroll. 1.* Like Polygons are in duplicate Ratio to their sides.

Coroll. 2. If three Lines are continually Proportional, the Polygon described on the first, shall have the same Ratio to the Polygon described on the second, as the first hath to the third; that is to say, in duplicate Ratio of that of the first Line to the second.

USE from the Learned Dr. Barrow.

By this is found a method of enlarging or diminishing a Right Lined figure in a Ratio given, as if you would have a Pentagon Quintuple of that Pentagon whereof CD is the side; then betwixt AB and 5 AB, find out a mean Proportional; upon this raise a Pentagon like unto that given, and it shall be quintuple also of the Pentagon given.

Hence also, if the Homologous sides of like figures be known, then will the Proportion of the figure be evident, viz. by finding out a third Proportional.

PROPOSITION XXI.

THEOREM.

Polygons which are like to a Third Polygon, are also like one to the other.

If Two Polygons are like to a third, they shall be like one to the other; for each of them may be divided into as many like Triangles as there is in the third. Now the Triangles which are like unto the same Third, are also like one to the other, because the Angles which are equal to a Third, are equal one to the other; and the Angles of the Triangles being equal, those of the Polygons of whom it is compounded, are so likewise.

I farther add that the sides of Triangles being in the same Ratio, those of the Polygons shall be so likewise, seeing they are the same.

PROPOSITION XXII. THEOREM.

Like Polygons described on four Lines which are Proportional, are also Proportional. And if the Polygons be in the same Ratio, the Lines on which they are described shall be so also.

If there be the same Ratio of BC to EF, as of HT to MN; there shall also be the same Ratio of the Polygon ABC to the like Polygon DEF, as of the Polygon HL to the like Polygon MO. Seek to the Lines BC, EF, a third Proportional G; and to the Lines HT, MN, a third Proportional P (by the 11th) seeing there is the same Ratio of BC to EF, as of HT to MN; and of EF to G, as of MN to P: there shall be by equality the same Ratio of BC to G, as of HT to P; and this Ratio shall be duplicate or doubled of the Ratio of BC to EF, or of HT to MN.

Demonstration. The Polygon ABC to the Polygon DEF, is in duplicate or doubled Ratio, of the Ratio of BC to EF (by the 21st) that is to say, as BC is

is to G; and the Polygon HL to MO, hath the same Ratio as HT to P. There is therefore the same Ratio of ABC to DEF, as of HL to MO.

And if like Polygons are proportional; the Lines being in sub-duplicate Ratio, shall be also Proportional.

U S E.

A, B, C, D,
3. 2. 6. 4.
9. 4. 36. 16
E, F, G, H.

THis Proposition may be easily applied to Numbers. If the Numbers A, B, C, are Proportional, their Squares EF, G, H, shall be so likewise; which we make use of in Arithmetick, and yet more use thereof in Algebra.

PROPOSITION XXIII.

THEOREM.

Equiangular Parallelograms have their Ratio compounded of the Ratio of their sides.

If the Parallelograms L and M be equiangular; the Ratio of L to M shall be compounded of that of AB to DE, and that of DB to DF. Joyn the Parallelograms in such a manner that their sides BD, DF, be on a streight Line, as also CD, DE; which may be, if they be equiangular. Compleat the Parallelogram BDEH.

M 4

De

Demonstrat. The Parallelogram L hath the same Ratio to the Parallelogram BDEH, as the Base AB hath to the Base BH or DE (*by the first*) the Parallelogram BDEH hath the same Ratio to the Parallelogram DFGE, that is to say M, as the Base BD hath to the Base DF. Now the Ratio of the Parallelogram L to the Parallelogram M, is compounded of that of L to the Parallelogram BDEH, and that of BDEH to the Parallelogram M. Thence the Ratio of L to M, is compounded of that of AB to DE, and of that of BD to EG. For example, if AB be eight, and BH five, BD four, DF seven; say as four is to seven, so is five to eight $\frac{5}{8}$; you shall have these three numbers eight, five, eight $\frac{5}{8}$; eight to five shall be the Ratio of the Parallelogram L to BDEH, the same as that of AB to DE; five to eight $\frac{5}{8}$ shall be that of the Parallelogram BDEH to M. So then taking away the middle term which is five, you shall have the Ratio of eight to eight $\frac{5}{8}$, for the Ratio compounded of both; or as 4 times 8 or 32, to 5 times 7 or 35.

PROPOSITION XXIV.

THEOREM.

IN all sorts of Parallelograms; those through which the Diameter passeth, are like to the greater.

Let the Diameter of the Parallelogram AC pass through the Parallelograms EF , GH ; I say they are like unto the Parallelogram AC .

Demonstration. The Parallelograms AC , EF , have the same Angle B ; and because in the Triangles BCD , IE is Parallel to the Base DC , the Triangles BEI , BCD , are equiangular. There is therefore (by the 4th) the same Ratio of BC to CD , as of BE to EI ; and consequently the sides are in the same Ratio. In like manner, IH is parallel to BC ; there shall then be the same Ratio of DH to HI , as of DC to BC ; and the Angles are also equal, all the sides being parallel: thence (by the 8th def.) the Parallelograms EF , GH , are like unto the parallelogram AC .

USE.

I Made use of this Proposition in the Fifth Proposition of the last Book of Perspective, to show that an Image was

M 5,

drawn

drawn like unto the Original, with the help of a Parallelogram composed of four Lines.

PROPOSITION XXV.

PROBLEM.

TO describe a Polygon like unto a given Polygon, and equal to any other Right-lined Figure.

If you would describe a Polygon equal to the Right-lined Figure *A*, and like unto the Polygon *B*; make a Parallelogram *CE* equal to the Polygon *B*. (by the 34th of the 1st) and on *DE*, make a Parallelogram *EF* equal to the Right-lined Figure *A* (by the 45th of 1st.) Then seek a mean Proportional *GH* between *CD* and *DF* (by the 13th.) Lastly, make on *GH* a Polygon *O*, like unto *B* (by the 18th.) It shall be equal to the Right-lined Figure *A*.

Demonstration. Seeing that *CD*, *GH*, *DE*, are continually Proportional, the Right-lined Figure *B* described on the first, shall be to the Right-lined Figure *O* described on the second, as *CD* is to *DE* (by the Coroll. of the 20th.) Now as *CD* is to *DE*, so is the Parallelogram *CE* to the Parallelogram *EF*, or as *B* to *A*, seeing they are equal. There is thence the same Ratio of *B* to *O*, as of *B* to

B to A. So then (by the 7th of the 5th.) A and O are equal.

U S E.

THis Proposition contains a change of Figures, keeping always the same Area, which is very useful, principally in Practical Geometry, do reduce an irregular figure into a Square.

PROPOSITION XXVI.

THEOREM.

IF in one of the Angles of a Parallelogram, there be described a lesser Parallelogram like unto the greater, the Diameter of the greater shall meet the Angle of the lesser.

If in the Angle D, of the Parallelogram AC; be described another lesser DG, like thereto: The Diameter DB shall pass through the Point G. For if it did not, but it passed through I, as doth the Line BID. Draw the Line IE parallel to HD:

Demonstration. The Parallelogram DI, is like unto the Parallelogram AC, (by the 24th.) Now it is supposed that the Parallelogram DG is also like thereto; thence the Parallelograms DI, DG, would be like; which is impossible: otherwise, there would be the same Ratio

Ratio of HI to IE or GF , as of HG to GF ; and (by the 7th of the 5th) the Lines HI , HG , would be equal.

The Twentieth, Twentieth, Twenty Eighth, and Twenty Ninth Propositions are unnecessary.

PROPOSITION XXX.

PROBLEM.

TO cut a Line given into extrem and mean Proportion.

It is proposed to cut the Line AB in extrem, and mean Proportion; that is to say, in such a manner, that there may be the same Ratio of AB to AC , as of AC to CB . Divide the Line AB (by the 11th of the 2d) in such a manner that the Rectangle comprehended under AB , CB , be equal to the Square of AC .

Demonstration. Seeing the Rectangle of AB , CB , is equal to the Square of AC ; there will be the same Ratio of AB to AC , as of AC to CB (by the 17th.)

USE.

THis Proposition is necessary in the Thirteenth Book of Euclid, to find the lengths of the Sides of some of the five Regular Bodies. Father Lucas of

of Sr. Sepulchres hath composed a Book
of the Proprieties of a Line, which is
cut into extreame and mean Proportion.

PROPOSITION XXXI
THEOREM.

A Polygon which is described on the
Base of a Right-angled Triangle,
is equal to the other two like Polygon de-
scribed on the other Sides of the same
Triangle.

If the Triangle ABC hath a Right-
Angle BAC , the Polygon D described
on the Base BC , is equal to the like
Polygons F , and E , described on the
Sides AB , AC .

Demonstration The Polygons D, E, F ,
are amongst themselves in duplicate
Ratio of their homologous Sides BC ,
 AC , AB , (by the 20th.) If there were
described a Square on those sides, they
would be also in duplicate Ratio to their
sides. Now (by the 47th of the 1st)
the Square of BC would be equal to
the Squares of AC , AB , thence the
Polygon D described on BC , is equal to
the like Polygons E , and F , described
on AB , AC .

U S E

This Proposition is made use of to
make all sorts of Figures greater or
lesser

lessor, for it is more universal than (the 47th. of the 1st) which notwithstanding is so useful, that it seemeth that almost all Geometry is established on this Principle.

The 32d Proposition is unnecessary.

PROPOSITION XXXIII.

THEOREM.

IN equal Circles, the Angles as well those at the Center as those at the Circumference, as also the Sectors are in the same Ratio as the Arks, which serve to them as Base.

If the Circles ANC , DOF , be equal; there shall be the same Ratio of the Angle ABC to the Angle DEF , as of the Ark AC to the Ark DF . Let the Ark AG , GH , HC , be equal Arks, and consequently aliquot parts of the Ark AC , and let be divided the Ark DF , into so many equal to AG , as may be found therein, and let Lines EI , EK , and the rest be drawn.

Demonstration. All the Angles AEG , GBH , HBC , DEI , IEK , and the rest, are equal (by the 37th of the third;) so then AG an Aliquot part of AC , is found in the Ark DF , as many times;

times as the Angle ABG an aliquot part of the Angle ABC , is found in DEF ; there is therefore the same Ratio of the Ark AC to the Ark DF , as of the Angle ABC to the Angle DEF . And because N and O are the halves of the Angles ABC , DEF , they shall be in the same Ratio as are those Angles; there is therefore the same Ratio of the Angle N to the Angle O , as of the Ark AC to the Ark DF .

It is the same with Sectors; for if the Lines AG , GH , HC , DI , IK , and the rest were drawn, they would be equal (by the 28th of the 3d;) and each Sector would be divided into a Triangle and a Segment. The Triangles would be equal (by the 8th of the 1st.) and the little Segments would be also equal (by the 24th of the 3d.) thence all those little Sectors would be equal; and so as many as the Ark BF containeth of aliquot parts of the Ark of AC , so many the Sector DKF would contain aliquot parts of the Sector AGC . There is therefore the same Reason of Ark to Ark, as of Sector to Sector.

The end of the Sixth Book.

THE
ELEVENTH BOOK
OF
Euclid's Elements :

THis Book comprehendeth the first Principles of Solid Bodies, whence it is impossible to establish any thing touching the third Species of quantity, without knowing what this Book teacheth us, which maketh it very necessary in the greatest part of the Mathematicks, in the first place, Theodosius's Sphericks, doth wholly suppose it, Spherical Trigonometry, the third part of Practical Geometry, several Propositions of Staticks, and of Geography, are established on the Principles of Solid Bodies, Gnomonicks, Conical Sections, and the Treatise of Stone-cutting, are not difficult, but because one is often obliged to represent on Paper, figures that are raised, and which
are

are comprised under several Surfaces. I leave out the Seventh, the Eighth, the Ninth, and the Tenth Books of Euclid's Elements, because they are unnecessary, in almost all the parts of the Mathematicks. I have often wondred that they have been put amongst the Elements, seeing it is evident that Euclid did compose them only to establish the Doctrine of incommensurables, which being only a Curiosity, ought not to have been placed with the Books of Elements, but ought to make a particular Treatise. The same may be said of the Thirteenth, and of the rest, for I believe that one may learn almost all parts of the Mathematicks, provided one understand well these Eight Books of Euclid's Elements.

DEFINITIONS.

1. **A** Solid is a Magnitude which hath length, breadth and thickness. Def. I.
Plate VII.
Fig. I.
As the Figure $LTNX$, its length is LT , its breadth NO , its thickness LN .
2. The extremities or terms of a Solid, are Surfaces.
3. A Line is at Right-angles, or Perpendicular to a Plane, when it is Perpendicular to all the Lines that it meeteth with in the Plane.

As

Fig. II.

As the Line AB shall be at Right-angles to the Plane CD , if it be Perpendicular to the Lines CD, FE ; which Lines being drawn in the Plane CD , pass through the Point B , in such sort that the Angles ABC, ABD, ABE, ABF , be Right.

4. A Plane is perpendicular to another, when the Line perpendicular to the common Section of the Planes, and drawn in the one, is also perpendicular to the other Plane.

Fig. III.

We call the common Section of the Planes, a Line which is in both Planes; as the Line AB , which is also in the Plane AC , as well as in the Plane AD . If then the Line DE , drawn in the Plane AD , and perpendicular to AB , is also perpendicular to the Plane AC ; the Plane AD shall be Right, or perpendicular to the Plane AC .

Fig. IV.

5. If the Line AB be not perpendicular to the Plane CD , and if there be drawn from the Point A , the Line AE perpendicular to the Plane CD , and then the Line BE , the Angle ABE , is that of the inclination of the Line AB , to the Plane CD .

Fig. V.

6. The inclination of one Plane to another, is the Acute Angle comprehended by the two perpendiculars to the common

common Section drawn in each Plane.

As the inclination of the Plane AB, to the Plane AD, is no other than the Angle BCD, comprehended by the Lines BC, CD, drawn in both Planes perpendicular to the common Section AE.

7. Planes shall be inclined after the same manner, if the Angles of inclination are equal.

8. Planes which are parallel being continued as far as you please, are notwithstanding equidistant.

9. Like solid figures, are comprehended, or terminated under like Planes equal in number.

10. Solid Figures, equal and like, are comprehended or terminated under like Planes equal both in Multitude and Magnitude.

11. A solid Angle is the concurrence Fig. VI.
or inclination of many Lines, which are in divers Planes. *As the concurrence of the Lines AB, AC, AD, which are in divers Planes.*

12. A Pyramid is a solid figure com- Fig. VI.
prehended under divers Planes, set upon one Plane, and gathered together to one Point. *As the Figure ABCD.*

13. A Prism is a solid figure contain- Fig. VII.
ed under Planes, whereof the two opposite are equal, like, and parallel, but the others

others are parallelograms. As the figure *AB* its opposite Planes may be Polygons.

14. A Sphere is a solid figure terminated by a single Superficies, from which drawing several Lines to a point taken in the middle of the figure, they shall be all equal; Some define a Sphere by the motion of a Semi-circle turned about its Diameter, the Diameter remaining immoveable.

15. The Axis of a Sphere is that unmoveable Line about which the semi-circle turneth.

16. The Center of the Sphere, is the same with that of the semi-circle which turneth.

17. The Diameter of a Sphere is any Line whatever, which passeth through the Center of the Sphere, and endeth in its Superficies.

18. A Cone is a figure made, when one Side of a Right-angled Triangle, viz. one of those that contain the Right Angle remaining fixed; the Triangle is turned round about till it return to the place from whence it first moved. And if the fixed Right Line be equal to the other which containeth the Right Angle, then the Cone is a Rectangled Cone, but if it be less, it is an Obtuse Angled Cone; if greater an Acute Angled Cone.

19. The

19. The Axis of a Cone is that fixed Line about which the Triangle is moved.

20. A Cylinder is a figure made by the moving round of a Right Angled Parallelogram, one of the Sides thereof, namely which contains the Right Angle, abiding fixed, till the parallelogram be turned about to the same place whence it began to move.

21. Like Cones and Cylinders, are those whose Axis and Diameters of their Bases are Proportional. Cones are right, when the Axis is perpendicular to the Plain of the Base, and they are said to be Scalene, when the Axis is inclined to the Base, and the Diameter of such Bases are in the same Ratio: We add, that inclined Cones to be like their Axis, must have the same inclination to the Planes of their Bases.

PROPOSITION I.

THEOREM.

A Strait Line cannot have one of its parts in a Plane, and the other without it. PLATE VII. Prop. I.

If the Line AB be in the Plane AD, it being continued, shall not go without, but all its parts shall be in the same Plane, For if it could be that BC were a part of AB continued. Draw in the Plane

Plane CD, the Line BD, perpendicular to AB: draw also in the same Plane, BE perpendicular to BD.

Demonstration. These Angles ABD, DBE, are both Right Angles; therefore (by the 14th of the first,) AB, BE, do make but one Line; and consequently BC, is not a part of the Line AB continued; otherwise two strait Lines CB, EB, would have the same part AB: that is AB would be part of both; which we have rejected as false in the Thirteenth Maxim of the first Book.

U S E

WE establish on this Proposition a principle in Cosmionicks, to prove that the shadow of the stile falleth not without the Plane of a great Circle, in which the Sun is. Seeing that the end or top of the stile is taken for the Center of the Heavens; and consequently for the Center of all the great Circles: the shadow being always in a straight Line, with the Ray drawn from the Sun to the Opaque Body; this Ray being in any great Circle, the shadow must also be therein.

PRO-

PROPOSITION II. THEOREM.

Lines which cut one another, are in the same Plane, as well as all the parts of a Triangle.

If the Two Lines BE, CD, cut one another in the Point A, and if there be made a Triangle by drawing the Base BC: I say that all the parts of the Triangle ABC, are in the same plane, and that the Lines BE, CD, are likewise therein.

Demonstration. It cannot be said that any one part of the Triangle ABC, is in a Plane, and that the other part is without; without saying that one part of a Line is in one Plane, and that the other part of the same Line is not therein; which is contrary to the first Proposition: and seeing that the sides of the Triangle are in the same Plane wherein the Triangle is; the Lines BE, CD, shall be in the same Plane.

USE.

This Proposition doth sufficiently determine a Plane, by two straight Lines mutually intersecting each other, or by a Triangle; I have made use thereof in Opticks, to prove that the objective paral.

parallel Lines which fall on the Tablet, ought to be Represented by Lines which concur in a Point.

PROPOSITION III. THEOREM.

The common section of two Planes is a straight Line.

IF Two Planes AB, CD, cut one another, their common section EF shall be a straight Line. For if it were not, take Two Points common to both Planes which let be E and F; and draw a straight Line from the point E to the point F, in the Plane AB, which let be EHF. Draw also in the Plane CD a straight Line from E to F; if it be not the same with the former, let it be EGF.

Demonstration. Those Lines drawn in the Two Planes are two different Lines, and they comprehend a space; which is contrary to the Twelfth Maxim. Thence they are but one Line, which being in both Planes, shall be their common section.

USE.

This Proposition is fundamental. We do suppose it in Gnomonicks, when we represent in a Dial, the Circles of the hours, marking only the common section of their Planes, and that of the Wall.

PRO.

PROPOSITION IV. THEOREM.

IF a Line be perpendicular to two other Lines which cut one another, it shall be also perpendicular to the Plane of those Lines.

If the Line AB be perpendicular to the Lines CD, EF, which cut one another in the point B; in such manner that the Angles ABC, ABD, ABE, ABF, be right, which a flat figure cannot represent; it shall be perpendicular to the Plane CD, EF; that is to say, that it shall be Perpendicular to all the Lines that are drawn in that Plane through the point B: as to the Line GBH. Let equal Lines be cut BC, BD, BE, BF; and let be drawn the Lines EC, DF, AC, AD, AE, AF, AG, and AH.

Demonstration. The four Triangles ABC, ABD, ABE, ABF, have their Angles Right in the Point B; and the Sides BC, BD, BE, BF, equal with the side AB common to them all. Therefore their Bases AC, AD, AE, AF, are equal (by the 4th of the 1st.)

2. The Triangles EBC, DBF, shall be equal in every respect, having the Sides BC, BD, BE, BF, equal; and the Angles CBE, DBF, opposite at the ver-

tex being equal; so then the Angles BCE, BDF, BFC, BFD, shall be equal (*by the 4th of the first,*) and their Bases EC, DF, equal.

3. The Triangles GBC, DBH, having their opposite Angles CBG, DBH, equal, as also the Angles BDH, BCG, and the sides BC, BD, they shall then have (*by the 26th of the 1st*) their Sides BG, BH, CG, DH, equal.

4. The Triangles ACE, AFD, having their sides AC, AD, AE, AF, equal, and the Bases EC, DF, equal, they shall have (*by the 8th of the 1st*) the Angles ADF, ACE, equal.

5. The Triangles ACG, ADH, have the Sides AC, AD, CG, DH, equal, with the Angles ADH, AGC: Thence they shall have their Bases AG, AH, equal.

Lastly, the Triangles ABH, ABG, have all their sides equal; thence (*by the 27th. of the 1st*) the Angles ABG, ABH, shall be equal, and the Line AB perpendicular to GH. So then the Line AB shall be perpendicular to any Line which may be drawn through the point B, in the Plane of the Lines CD, EF, which I call perpendicular to the Plane.

USE.

U S E.

THis Proposition cometh often in use in the first Book of Theodosius; for example, to demonstrate that the Axis of the World is perpendicular to the Plane of the Equinoctial. In like manner in Gnomonicks, we demonstrate by this Proposition, that the Equinoctial is perpendicular to the Meridian in Horizontal Dials, it is not less useful in other Treatises; as in that of Astrolabes, or in the Sections of Stone.

PROPOSITION V.

THEOREM.

IF a Line be perpendicular to three other Lines which cut one the other in the same Point, these three Lines shall be in the same Plane.

If the Line AB be perpendicular to three Lines BC, BD, BE, which cut one another in the Point B, the Lines BC, BD, BE, are in the same Plane. Let the Plane AE be the Plane of the Lines AB, BE; and let CF be the Plane of the Lines BC, BD. If BE was the common Section of the two Planes, BE would be in the Plane of the Lines BC, BD, as we pretend it

should. Now if BE be not the common Section ; let it be BG.

Demonstration. AB is perpendicular to the Lines BC, BD ; it is then perpendicular to the Plane CF (*by 4th and 5th Def.*) AB shall be also perpendicular to BG. Now it is supposed that it is perpendicular to BE ; thence the Angles ABE, ABG, would be right, and equal, and notwithstanding the one is a part of the other. So then the two Planes cannot have any other common Section besides BE ; it is therefore in the Plane CF.

PROPOSITION VI.

THEOREM.

The Lines which are perpendicular to the same Plane, are parallel.

If the Lines AB, CD, be perpendicular to the same plane EF ; they shall be parallel. It is evident that the internal Angles ABD, BDC, are right ; but that is not sufficient, for we must also prove that the Lines AB, CD, are in the same Plane. Draw DG, perpendicular to BD, and equal to AB : Draw also the Lines BG, AG, AD.

Demonstration. The Triangles ABD, EDG, have the sides AB, DG, equal : BD is common ; the Angles ABD, BDG,

BDG, are right. Thence the Bases AD, BG, are equal (*by the 4th of the 1st.*) Moreover the Triangles ABG, ADG, have all their sides equal, thence the Angles ABG, ADG, are equal; and ABG being right, seeing AB is perpendicular to the Plane, the Angle ADG is right. Therefore the Line DG is perpendicular to the three Lines CD, DA, DB, which by consequence are in the same Plane (*by the 5th.*) Now the Line AB is also in the Plane of the Lines AD, DC, (*by the second;*) thence AB, CD, are in the same place.

Coroll. Two Lines which are parallel are in the same place.

U S E.

WE demonstrate by this Proposition, that the hour Lines are parallels amongst themselves, in all Planes which are parallel to the Axis of the World; as in the Polar and Meridian Dials, and others.

PROPOSITION VII.

THEOREM.

A Line which is drawn from one parallel to another, is in the same Plane which those Lines are.

Observe that the Line is in the same Plane as the two parallel Lines.

The Line CB being drawn from the point B of the Line AB, to the point C of its parallel CD : I say that the Line CB is in the Plane of the Lines AB, CD.

Demonstration. The parallels AB, CD, are in the same Plane; in which if you draw a strait Line from the point C to the point B, it will be the same with CB; otherwise two strait Lines would inclose a space, *contrary to the 12th Maxim.*

PROPOSITION VIII. THEOREM.

IF of two parallel Lines, the one be perpendicular to a Plane, the other shall be so likewise.

As if the two parallel Lines AB, CD; the one AB be perpendicular to the Plane EF: CD shall be so likewise. Draw the Line BD, seeing that ABD is a right Angle, and that the Lines AB, CD are supposed parallel; the Angle CDB shall be right (*by the 30th of the 1st.*) Wherefore if I demonstrate that the Angle CDG is right, I shall prove (*by the 4th*) that CD is perpendicular to the Plane EF. Make the right Angle BDG, and make DG equal to AB, then draw the Lines BG, AG.

De.

Demonstration. The Triangles ABD, BDG, have their sides AB, DG, equal; the side BD is common, the Angles ABD, BDG, are right: Thence (*by the 4th of the 1st*) the Bases AD, BG, are equal. The Triangles ADG, ABG, have all their sides equal; so then (*by the 6th*) the Angles ADG, ABG, are equal. This last is right, seeing the Line AB is supposed perpendicular to the Plane EF; thence the Angle ADG, is right; and the Line DG being perpendicular to the Lines DB, DA, it shall be perpendicular to the Plane of the Lines AD, DB; which is the same in which are the parallels AB, CD. So then the Angle GDC is a right Angle (*by the 5th Def.*)

PROPOSITION IX.

THEOREM.

The Lines which are parallel to a third, are parallel amongst themselves.

If the Lines AB, CD, are parallel to the Line EF; they shall be parallel to each other, although they be not all three in the same Plane. Draw in the Plane of the Lines AB, EF, the Line HG perpendicular to AB, it shall be also perpendicular to EF (*by the 29th*

of the 1st.) In like manner draw in the Plane of the Lines EF, CD, the Line HI perpendicular to EF, CD.

Demonstration. The Line EH being perpendicular to the Lines HG, HI: is also perpendicular to the Planes of the Lines HG, HI, (by the 4th) thence (by the 8th) the Lines AG, CI, are so likewise, and (by the 6th) they shall be parallel.

U S E.

THis Proposition serveth very often in Prospective, to determine on the Plane the Image of the parallel Lines, and in the sections of Stones, where we prove that the sides of the Squares are parallel to each other, because they are so to some Line in a different Plane. In Gnomonicks we prove that vertical Circles ought to be drawn by perpendiculars on the Wall, because that the Lines which are their common Section with the Wall, are parallel to the Line drawn from the Zenith to the Nadir.

PROPOSITION X.

THEOREM.

IF two Lines which meet in a point are parallel to two other Lines in a different Plane; they will form an equal Angle. If

If the Lines AB , CD ; AE , CF , are parallel, although they be not all four in the same Plane; the Angles BAE , DCF , shall be equal. Let the Lines AB , CD ; AE , CF , be equal: and draw the Lines BE , DF , AC , BD , EF .

Demonstration. The Lines AB , CD , are supposed parallel and equal; thence (by the 33d of the 1st) the Lines AC , BD , are parallel and equal; as also AC , and EF ; and (by the preceding) BD , EF , shall be also parallel and equal; and (by the 33d of the first) BE , DF , shall be parallel and equal. So then the Triangles BAE , DCF , have all their sides equal; and (by the 8th) the Angles BAE , DCF , shall be equal.

Coroll. One might make some Propositions like unto this, which would not be unuseful; as this. If one should draw in a parallel Plane, the Line CD parallel to AB , and if the Angles BAC , DCF , are equal, the Lines AC , CF , are parallel.

U S E.

WE demonstrate by this Proposition, that the Angles which are made by the four Circles in a Plane parallel

parallel to the Equator, are equal to those which they make in the Plane of the Equator.

PROPOSITION XI.

PROBLEM.

TO draw a perpendicular to a Plane, from a point given without the Plane.

If you would from the point C, draw a perpendicular to the Plane AB, draw the Line EF at discretion, and CF perpendicular thereto (by the 12th of the first), then draw (by the 11th of the 1st) in the Plane AB, the Line FG perpendicular to ED, and CG perpendicular to FG. I demonstrate that CG shall be perpendicular to the Plane AB. Draw GH parallel to EF.

Demonstration. The Line EF being perpendicular to the Lines CF, FG, it shall be perpendicular to the Plane CFG (by the 4th) and HG being parallel to EF, shall be also perpendicular to the same Plane (by the 8th) and seeing that CG is perpendicular to the Lines GF, GH, it shall be perpendicular to the Plane AB (by the 4th.)

PRO-

PROPOSITION XII.

PROBLEM.

TO draw a perpendicular to a Plane, from a point of the same Plane.

To draw from the point C a perpendicular to the Plane AB; draw from the Point E taken at discretion without the Plane, ED perpendicular to the same Plane (by the 11th.) Draw also (by the 30th of the 1st) CF parallel to DE, CF shall be perpendicular to the Plane AB (by the 8th.)

PROPOSITION XIII.

THEOREM.

THere cannot be drawn from the same point two Perpendiculars to a Plane.

If the two Lines CD, CE, drawn from the same point C, were perpendicular to the Plane AB; and that CE was the common Section of the Plane of those Lines, with the Plane AB; the Angles ECF, DCF, would be right; which is impossible.

I further add, that there cannot be drawn from the same point D two perpendiculars DC, DF, to the same Plane AB: for having drawn the Line CF, there will be made two Right Angles.

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Angles DCF, DFC, in one Triangle
contrary to the 3rd of the 1st.

U S E.

THis Proposition is necessary to shew,
that a Line which is drawn perpen-
dicular to a Plane is well determined,
seeing there can be drawn but only one
from a Point.

PROPOSITION XIV.
THEOREM.

Planes are parallel to which the same
Line is perpendicular.

If the Line AB be perpendicular to
the Planes AC, BD, they shall be pa-
rallel; that is to say, they shall be in
all places equidistant. Draw the Line
DC parallel to AB (by the 30th of the
1st) and joyn the Lines BD, AC.

Demonstration. It is supposed that
AB is perpendicular to the Planes AC,
BD; thence the Line CD, which is
parallel thereto, shall be also perpendi-
cular to them (by the 8th;) so then the
Angles B and D, A and C; shall be
Right; and (by the 29th of the 1st) the
Lines AC, BD, shall be parallel. The
figure ABCD shall be thence a pa-
rallelogram. Now (by the 34th of the
1st) the Lines AB, CD, are equal;
that

that is to say, that the Planes in the Points A and B, C and D, are equi-distant : So then because we can draw the Line CD from any Point whatever, the Planes AC, BD, shall be equi-distant in all places.

U S E.

Theodosius demonstrateth by this, that the Circles which have the same Poles, as the Equinoctial, and the Tropicks, are parallel ; because the Axis of the World is perpendicular to their Planes.

PROPOSITION XV.

THEOREM.

IF two Lines which meet in the same point are parallel to two Lines of another plane ; the Planes of those Lines shall be parallel.

If the Lines AB, AC, in the one Plane, are parallel to the Lines DF, DE, which are in another Plane ; the Planes BC, FE, are parallel. Draw AI perpendicular to the Plane BC (by the 11th) and GI, IH, parallel to FD, DE ; they shall also be parallel to the Lines AB, AC, (by the 9th.)

Demonstration. The Lines AB, GI, are parallel, and the Angle IAB is Right, seeing AI is perpendicular to the

the Plane BC: thence (by the 29th of the 1st) the Angle AIG is right. AIH is also right: Thence (by the 4th) the Line AI is perpendicular to the Plane GH, and it being also perpendicular to the Plane BC, the Planes BC, FE, shall be parallel (by the 14th.)

PROPOSITION XVI.
THEOREM.

IF a Plane cut two Planes which are parallel, its common Sections with them shall be parallel Lines.

If the Plane AB cutteth two other parallel Planes AC, BD: I demonstrate that the common Sections AF, BE, shall be parallels. For if they were not, they would meet if continued, for example in the Point G.

Demonstration. The Lines AF, BE, are in the Planes AC, BD, and do not go out of the same (by the 1st) wherefore if they meet each other in G, the Planes would meet each other likewise, and consequently they would not be parallel, contrary to what we have supposed.

U S E.

WE demonstrate by this Proposition, in the Treatise of Conick Sections and

and Cylindricks; that the Cone or Cylinder being cut by a parallel to its Base, the Sections are Circles; we describe Astrolabes; We prove in Gnomonicks, that the Angles the hour Circles make with a Plane parallel to a great Circle, are equal to those which they make with the Circle it self; we demonstrate in Perspective, that the Image of the object's Lines which are perpendicular to the Tablet, concur in the point of vision.

PROPOSITION XVII.

THEOREM.

Two Lines are divided proportionally by parallel Planes.

Let the Lines AB , CD be divided by parallel Planes; I say that AE , EB , and CF , FD , are in the same Ratio. Draw the Line AD , which meeteth the Plane EF in the point G ; draw also AC , BD , FG , EG .

Demonstration. The Plane of the Triangle ABD , cutteth the three Planes; thence (by the 16th) the Sections BD , EG , shall be parallel; and (by the 2d of the sixth.) there shall be the same Ratio of AE to EB , as of AG to GD . In like manner, the Triangle ADC cutteth the Planes EF , AC ; thence the Sections AC , GF , are parallel; and there shall
be

be the same Ratio of FC to FD, as of AG to GD, that is to say, as of AE to EB.

PROPOSITION XVIII.

THEOREM.

IF a Line be perpendicular to a Plane, all the Planes in which that Line shall be found, shall be perpendicular to the same Plane.

If the Line AB be perpendicular to the Plane ED, all the Plains in which it shall be found, shall be also perpendicular to ED. Let AB be in the Plane AE, which hath for common section with the Plane ED, the Line BE, unto which let there be drawn the perpendicular FI.

Demonstration. The Angles ABI, BIF, are Right; thence the Lines AB, FI, shall be parallel; and (by the 8th) FI shall be perpendicular to the Plane ED. So then the Plane AE shall be perpendicular to the Plane ED, (by the 5th. Def.)

U S E.

THe first Proposition in Gnomonicks; and which may stand for a fundamental, is established on this Proposition, it is also often made use of in Spherical Tri-

Trigonometry, in Perspective, and generally in all Treatises, which are obliged to consider several Planes.

PROPOSITION XIX.
THEOREM.

IF Two Planes which each cut other, are perpendicular to another, their common section shall also be perpendicular thereto.

If the Two Planes AB, ED, cut each other and are perpendicular to the Plane IK; there common section EF is perpendicular to the Plane IK.

Demonstration. If EF be not perpendicular to the Plane IK; let there be drawn in the Plane AB, the Line GF perpendicular to the common section BF; and seeing the Plane AB is perpendicular to the Plane IK; the Line GF, shall be perpendicular to the same Plane. Let there also be drawn FH perpendicular to the common section DF, it shall be perpendicular to the Plane IK. Thus we shall have Two perpendiculars to the same Plane, drawn from the same point F, *contrary to the 13th. Proposition.* We must therefore conclude that EF is perpendicular to the Plane IK.

USE.

U S E.

WE demonstrate by this Proposition, that the Circle which passeth through the Poles of the World, and through the Zenith, is the Meridian, and cutteth in two equally all the Diurnal Arks, and that every star employs the same space of time from their rising to this Circle, as they do from this Circle to their setting.

PROPOSITION XX.

THEOREM.

IF three Plain Angles compose a solid Angle, any Two of them shall be greater than the third.

If the Angles BAC, BAD, CAD, compose the solid Angle A; and if BAC be the greatest of them; the other Two BAD, CAD, taken together, are greater than BAC. Let CAE be equal to CAD, and let the Lines AD, AE, be equal. Draw the Lines CEB, CD BD.

Demonstration. The Triangles CAE, CAD, have the sides AD, AE, equal; CA, common; and the Angles CAD, CAE, equal; thence (by the 4th. of the first,) the Bases CD, CE, are equal.

Now

Now the Sides CD , BD , are greater than the side BC (*by the 20th of the first*) wherefore taking away the equal Lines CD , CE ; the Line BD shall be greater than BE . Moreover the Triangles BAE , BAD , have their sides AD , AE , equal; AB common; and the Base BD greater than EB : thence (*by the 18th. of the first;*) the Angle BAD is greater than the Angle BAE ; and adding the Angles CAD , CAE ; the Angles BAD , CAD , shall together be greater than BAE , CAE , that is to say, than CAB .

PROPOSITION XXI.

THEOREM.

ALL the Plain Angles which Compose a solid Angle are less than four Right.

If the Plane Angles BAC , CAD , BAD , Compose the solid Angle A ; they shall be less than four Right. Draw the Lines BC , CD , BD , and you shall have a Pyramid, which hath for Base the Triangle BCD .

Demonstration. There is made a solid Angle at the point B , of which the Angles ABC , ABD , taken together, are greater than the Angle CBD at the Base. In like manner ACB , ACD , are
grea-

greater than BCD; and the Angles ADC, ADB, are greater than CDB. Now all the Angles at the Base CDB, are equal to Two Right; thence the Angles ABC, ABD, ACB, ACD, ADC, ADB, are greater than Two Right. And because all the Angles of the Three Triangles BAC, BAD, CAD, are equal to fix Right; in taking away more than Two Right, there will remain less than four Right, for the Angles which are made at the point A. If the solid Angle A was Composed of more than Three plain Angles; in such manner that the Base of the Pyramid was a Polygon; it might be divided into Triangles: and by making the supputation, one would always find that all the Plane Angles which from the solid Angle, taken together, are less than four Right.

U S E.

Those Two Propositions serve to determine, when of several plain Angles there may be made a solid Angle; which is often necessary in the Treatise of cutting of stone, and in the following Propositions.

The Two and Twentieth and Three and Twentieth Propositions are needless.
PRO-

PROPOSITION XXIV.

THEOREM.

IF a solid Body be terminated by parallel Planes, the opposite Planes shall be like Parallelogram, and equal.

If the solid AB be terminated by parallel Planes, the opposite Superficies shall be like and equal parallelograms.

Demonstration. The parallel Planes AC, BE, are cut by the Plane FE; wherefore the common sections AF, DE, are parallel (*by the 15th.*) In like manner DF, AE, whence AD shall be a parallelogram. I demonstrate after the same manner, that AG, FB, CG, and the rest, are parallelogram. I further add, that the opposite parallelograms; for example AG, BF, are like and equal: The Lines AE, EG, are parallel to the Lines FD, BD, and also equal; thence the Angles AEG, FDB, are equal (*by the 15th.*) I may also demonstrate that all the sides, and all the Angles of the opposite parallelograms are equal. Thence the Parallelograms are like and equal,

PRO-

The Eleventh Book of
PROPOSITION XXV.
T H E O R E M.

IF a parallelepipedon be divided by a Plane parallel to one of its sides; the Two solids which result from this Division, shall be in the same Ratio as their Bases.

If the parallelepipedon AB be divided by the Plane CD parallel to the opposite Planes AF, BE, the solid AC to BD, shall be in the same Ratio as the Base AI to the Base DG. Let us imagine that the Line AH, which represents the height of the figure is divided into as many parts as one pleaseth; for example in Ten Thousand, which we may take to be indivisible, that is to say, without thinking they can be divided. Let us also suppose or imagine as many Surfaces parallel to the Base AG, as there is parts in the Line AH: I express but one for all, as OS; in which manner the Solid AB may be Composed of all those Surfaces of equal thickness, as would be a Ream of Paper composed of all its Sheets, when laid on each other. It is evident that then the Solid AC shall be composed of Ten Thousand Surfaces equal to the Base AI, (*by the preceding;*) and the Solid BD shall contain Ten Thousand Surfaces equal to the Base DG.

De

Demonstration. Each Surface of the Solid AC, hath the same Ratio to each Surface of the Solid BD, as the Base AI hath to the Base DG; seeing each of them are equal to their Bases; thence (*by the 12th of the 5th*) all the Surfaces of the Solid AC taken together, shall have the same Ratio to all the Surfaces of the Solid DB; as hath the Base AI to the Base DG. Now all the Surfaces of the Solid AC, compose the Solid AC, which hath no other parts but its Surfaces, and all the Surfaces of the Solid DB, are nothing else but the Solid DB; thence the Solid AC to the Solid DB, hath the same Ratio as hath the Base AI to the Base DG.

U S E.

THis way of Demonstration is that of Cavallerius: I find it to be very clear, provided it be made use of as it ought, and that the Line which is the measure of the thickness of its Surfaces, be taken in the same manner in the one and in the other Term. I will make use thereof hereafter, to make more easy some Demonstrations which otherwise are too incumbred.

PRO.

PROPOSITION XXVI
THEOREM.

A *Parallelepipedon is divided into two equally, by a Diagonal Plane.*

Let the **Parallelepipedon** **AB** be divided by the **Plane** **CD**, drawn from one Angle to the other: I say that it divideth the same into two equally. Let the **Line** **EA**, be supposed to be divided into as many parts as one listeth; and that from each, one hath drawn as many **Planes** parallel to the **Base** **AF**; each of those **Planes** is a **Parallelogram** equal to the **Base** **AF** (*by the 24th.*)

Demonstration. All the **Parallelograms** which can be drawn parallel to the **Base** **AF**, are divided into two equally by the **Plane** **CD**; for the **Triangle** which will be made on each side of the **Plane** **CD**, have their common **Base** equal to **CG**; and their sides equal, seeing they are those of a **parallelogram**. Now it is evident that the **Parallelepipedon** **AB**, containeth nothing else but its **parallelogram** **Surfaces**, each of which is divided into two equal **Triangles**: therefore the **parallelepipedon** **AB**, is divided into two equally by the **Plane** **CD**.

The

The twenty seventh and twenty eighth Propositions are useless, according to this way of Demonstration.

PROPOSIT. XXIX. XXX. & XXXI.

THEOREM.

Parallelepipedons having the same height and the same, or equal Bases, are equal.

If the Parallelepipedon CD, AB, have equal perpendicular height AE, FG, with equal Bases AH, CI, on the same, they shall be equal. Let be put the two Bases AH, CI, on the same Plane, seeing their heights are equal, the Bases EB, FD, shall be in the same Plane, parallel to that of the Bases AH, CI. Let it be imagined that the perpendicular FG or EA, is divided into as many equal parts as one listeth; for example into ten thousand; and there be drawn through each Surfaces or Planes of the same thickness; to express which I mention but one, for all, which shall be KM. Each Surface shall form in each Solid a Plane parallel like and equal its Base. (by the 24th) as KL, OM, and there shall be as many of them in the one as in the other; seeing their thickness which I take perpendicularly

O

in

in the Line of their height, is equal.

Demonstration. There is the same Ratio of the Base AH to the Base CI, as of each plain KL to OM: Now there is the same number of them in the one as there is in the other, so then (by the 15th of the 5th) there shall be the same Ratio of all the Antecedents to all the Consequents, that is to say of all the Solid AB to all the Solid CD, as of the Base AH to the Base CI. Now it is supposed that the Bases are equal: thence the Solids AB, CD, are equal.

Coroll. To find the Solidity of a Parallelepipedon, we multiply the Base by its height taken perpendicularly, because that perpendicular sheweth how many times their is found Surfaces equal to the Base. As if I take a foot for an indivisible measure, that is to say, which I will not conceive subdivided: If the Base should contain twelve foot Square, and that the perpendicular height were ten foot; I should have one hundred and twenty Cubick feet, for the Solidity of the Body AB. For seeing the height AE, is ten foot; I may make ten parallelograms equal to the Base, and which shall have each of them one foot thickness. Now the Base with one foot thickness, makes twelve foot

foot Cubick; it shall then contain One hundred and twenty, if it hath ten foot height.

PROPOSITION XXXII. THEOREM.

P*arallelepipedons of the same height are in the same Ratio as their Bases.*

I have demonstrated this Proposition in the precedent, in proving that there is the same Ratio of the Parallelepipedon AB to the Parallelepipedon CD, as of the Base AH to the Base CI.

Coroll. Parallelepipedons which have equal Bases, are in the same Ratio as are their height; as the Parallelepipedons AB, AL, which have for their Perpendicular height AK, AE; for if the height AK were divided into as many aliquot parts as one would, and AE into as many equal parts of the first, as it would contain, and if there were drawn through each as many Planes parallel to the Base, I say as many as AE would contain of aliquot parts of AK, so many Solids AB shall contain, of Surfaces equal to the Base, which are aliquot parts of the Solid AL. Therefore (by the 5th of the 5th) there shall be the same Ratio of the Solid AB to the Solid AL, as of the height to the height AK.

U S E.

THe three precedent Propositions contain almost all the measuration of Parallelepipedons, and serve as the first Principles in this matter. It is thus we measure the Solidity of a Wall, by multiplying their Bases by their heights.

PROPOSITION XXXIII.

THEOREM.

Like Parallelepipedons are in tripled proportion of their homologous sides. If the Parallelepipedons AB, CD, are like; that is to say, if all the Planes of the one are like unto all the Planes of the other; and if all their Angles are equal, in such manner that they may be placed in a strait Line, that is to say, that AE, EF; HE, EI; GE, EC; be strait Lines, and that there is the same Ratio of AE to EF, as of HE to EI; and of GE to EC. I demonstrate that four Solids are continually proportionals, according to the Ratio of EA, to that which is homologous thereto, which is EF or DI.

Demonstration. The Parallelepipedon AB, hath the same Ratio to EL of the same height, as hath the Base AH to the Base EO (by the 32d.) Now the Base
AH

AH to the Base EO, hath the same Ratio as AE to EF (*by the 1st of the 6th.*) In like manner the Solid EL to the Solid EK, is the same as that of the Base EO to the Base ED, that is to say, as of HE to EI. In fine the Solid EK to the Solid EN, hath the same Ratio, as hath the height GE to the height EC (*by the Coroll.*) or (taking EF for their common height) as of the Base GI to the Base CI, that is to say, as of GE to EC. Now the Ratio of AE to EF, and of HE to EI, and of GE to EC, is the same, as we did suppose: By consequence, there is the same Ratio of AB to EL, as of EL to EK, and of EK to CD. Therefore (*by the 11th def. of the 5th*) the Ratio of AB to CD, shall be triple of that of AB to EL, or of AE to his homologous EF.

Coroll. If four Lines are continually proportional, the Parallelepipedon described on the first, hath the same Ratio to a like Parallelepipedon described on the second, as the first hath to the fourth, for the Ratio of the first to the fourth is tripled of that of the first to the second.

U S E.

YOU may comprehend by this Proposition, that the Celebrated Proposition of the Duplication of the Cube, proposed by the Oracle, consisteth in this, to find two means continually proportional. For if you propose for this first term, the side of the first Cube; and that the fourth term be the double of this first: If you find two mean proportionals, the Cube described on the first Line shall have the same Ratio to that described on the second, as the first Line hath to the fourth, which shall be as one to two. We also correct by this Proposition the false opinion of those which imagine, that like solids are in the same Ratio as are their sides, as if a Cube of one foot long were the half of a Cube of two foot long, altho' it be but the 8th part thereof. This is the Principle of the Calibres, which may be applied not only to Cannon Shot, but also to all like Bodies. For example, I have seen a person which would make a Naval Architecture, and would keep the same Proportions in all sorts of Vessels: but argued thus; if a Ship of one hundred Tuns, ought to be fifty foot long, a Ship of two hundred ought to be one hundred foot by the Keel. In which he was much mistaken, for instead of making a
Ship

Ship of twice that Burden, he would make one Octuple. He ought to have given to the second Ship to be double in Burden to the first, somewhat less than sixty three foot.

PROPOSITION XXXIV.

THEOREM.

Equal Parallelepipedons have their Bases and height Reciprocal, and those which have their Bases and height Reciprocal, are equal.

If the Parallelepipedons AB, CD, are equal, they shall have their Bases and height reciprocal; that is to say, there shall be the same Ratio of the Base AE to the Base CF, as of the height CH to the height AG. Having made CI equal to AG, draw the Plane IK parallel to the Base CF.

Demonstration. The Parallelepipedon AB, hath the same Ratio to CK of the same height, as hath the Base AE to CF, (by the 32d.) Now as AB to CK, so is CD to the same CK, seeing that AB and CD are equal; and as CD is to CK, which hath the same Base, so is the height CH to the height CI (by the Coroll. of the 32d.) Wherefore, as the Base AE is to the Base CF, so is the height CH to the height CI or AG.

I further add, that if there be the same Ratio of AE to CF, as of the height CH to the height AG; the Solids AB, CD, shall be equal.

Demonstration. There is the same Ratio of AB to CK, of the same height, as of the Base AE to the Base CF (*by the 32d*) there is also the same Ratio of the height CH to the height CI or AG, as of CD to CK; we suppose that the Ratio of AE to CF, is the same of that of CH to CI or AG; so then there shall be the same Ratio of the Solid AB to the Solid CK, as of the Solid CD to the same Solid CK. Therefore (*by the 8th of the 5th*) the Solids AB, CD, are equal.

U S E.

THis reciprocation of Bases, rendereth these Solids easy to be measured; it hath also a kind of Analogy with the sixteenth Proposition of the Sixth Book which beareth this, that parallelograms equiangled and equal, have their Sides reciprocal, and it demonstrateth also as well the practice of the Rule of Three.

PRO.

PROPOSITION XXXVI. THEOREM.

IF three Lines are continually proportional, the Solid Parallelepipedon made of them is equal to an equiangular Parallelepipedon, which hath all its sides equal to the middle Line.

If the Lines A, B, C, be continually proportional, the parallelepipedon FE made of them, that is to say, which hath the side FI equal to the Line A, and HE equal to B, HI equal to C; is equal to the equiangular Parallelepipedon KL, which hath its sides LM, MN, KN equal to the Line B. Let there be drawn from the Points H and N, the Lines HP, NQ, perpendicular to the Planes of the Bases; they shall be equal, seeing that the Solid Angles E and K are supposed equal (in such manner that if they did penetrate each other, they would not surpass each other) and that the Lines EH, KN, are supposed equal. Therefore the heights HP, NQ are equal.

Demonstration. There is the same Ratio of A to B, or of FI to LM, as of B to C, or of LM to HI; so then the parallelogram FH comprehended under FI, IH, is equal to the paralle-

logram LN, comprehended under LM, MN, equal to B (by the 15th of the 6th) the Bases are thence equal. Now the heights HP, NQ, are so likewise; therefore (by the 31st) the Parallelepipedons are equal.

PROPOSITION XXXVII. THEOREM.

IF four Lines are proportional, the Parallelepipedons described on those Lines shall be proportional, and if like Parallelepipedons are proportional, their homologous sides shall be so likewise.

If A to B be in the same Ratio as C to D; the like Parallelepipedons which shall have for their homologous sides A, B, C, D, shall be in the same Ratio.

Demonstration. The Parallelepipedon A to the Parallelepipedon B, is in triple Ratio of that of the Line A to the Line B, or of that of the Line C to the Line D. Now the Parallelepipedon C to the Parallelepipedon D, is also in triple Ratio of that of C to D (by the 33d) there is therefore the same Ratio of the Parallelepipedon A to the Parallelepipedon B, as of the Parallelepipedon C to the Parallelepipedon D.

PRO.

PROPOSITION XXXVIII.
THEOREM.

IF two Planes be perpendicular to each other, the perpendicular drawn from one Point of one of the Planes to the other, shall fall on the common Section.

If the Planes AB, CD, being perpendicular to each other, there be drawn from the Point E of the Plane AB, a Line perpendicular to the Plane CD; it shall fall on AG, the common Section of the Planes. Draw EF perpendicular to the common Section AG.

Demonstration. The Line EF perpendicular to AG, the common Section of the Planes, which we supposed perpendicular, shall be perpendicular to the Plane CD (by the third def.) and being there cannot be drawn from the Point E two perpendiculars to the Plane CD (by the 13th) the perpendicular shall fall on the common Section AG.

USE.

This Proposition ought to have been after the 17th, seeing it hath respect to Solids in general. It is of use in the Treatise of Astrolabes, to prove that in the Analemma, all the Circles which are perpendicular to the Meridian, are Great Lines.

PRO.

PROPOSITION XXXIX.
THEOREM.

IF there be drawn in a parallelepipedon two Planes dividing the opposite sides equally; their common Section and the Diameter doth also cut each other equally.

Let the opposite sides of the parallelepipedon AB , be divided into two equally by the Planes CD , EF ; their common Section GH ; and the Diameter BA shall divide each other equally in O . Draw the Lines BG , GK , AH , LH . I prove in the first place, that they make but one strait Line, for the Triangles DBG , KMG , have their sides BD , KM , equal, seeing they are the halves of equal sides; as also GD , GM . Moreover, DB , KM , being parallel, the alternate Angles BDG , GMK , shall be equal (by the 29th of the first;) so then (by the 4th of the first) the Triangles DBG , KGM , shall be equal in every respect; and consequently the Angles BGD , KGM . Now (by the 16th of the 1st) BG , GK , is but one Line, as also LH , HA ; thence AL , BK , is but one Plane, in which is found the Diameter AB and GH , the common Section of the Plane. The Plane AL , BK , cutting the parallel
Planes

Planes AN, CD, shall have its common Sections GH, AK, parallel, and (by the 4th of the 6th) there shall be the same Ratio of BK to GK, as of BA to BO; and of AK to OG (by the 4th of the 6th.) Now BK is double to BG; thence BA is double to BO, as AK equal to HG, is double to GO. So then the Lines GH, AB, cut each other equally in the Point O.

Coroll. 1. All the Diameters are divided in the Point O.

Coroll. 2. Here may be put Corollaries which depend on several Propositions; for example, that Triangular Prisms of equal height, are in the same Ratio as are their Bases, for the Parallelepipedons of which they are the halves (by the 32d) are in the same Ratio as their Bases, so that the half Bases and the half parallelepipedons, that is to say the Prisms, shall be in the same Ratio.

Coroll. 3. Polygon Prisms of the same height, are in the same Ratio as are their Bases; seeing they may be reduced to Triangular Prisms, which shall each of them be in the same Ratio as their Bases.

Coroll. 4. There may be applied to the Prism the other Propositions of the parallelepipedons; for example, that equal

equal Prisms have their heights and Bases Reciprocal, that like Prisms are in triple Ratio of their homologous sides.

U S E.

This Proposition may serve to find the Center of gravity of Parallelepipedons, and to Demonstrate some Propositions of the Thirteenth and Fourteenth Books of Euclid.

PROPOSITION XL.
THEOREM.

That Prism which hath for its Base a Parallelogram double to the Triangular Base of another, and of the same height, is equal thereto.

Let there be proposed Two Triangular Prisms ABE , CDG , of the same height; and let be taken for the Base of the first the Parallelogram AE , double to the Triangle FGC , the Base of the second Prism. I say that those Prisms are equal. Imagine that the Parallelepipedons AH , GI , are drawn.

Demonstration. It is supposed that the Base AE is double to the Triangle FGC : Now the Parallelogram GK being also double to the same Triangle FGC , the Parallelograms

quants AE, GK, are equal: and consequently the Parallelopipedons AH, GI, which have their Bases and Heights equal, are equal; and Prisms which are the halves (by the 18th) shall be also equal.

The End of the Eleventh Book.

THE
TWELFTH BOOK
OF

Euclid's Elements:

EUCLID, after having given in the preceding Books, the general Principles of Solids, and explained the way of measuring the most Regular, that is to say those which are terminated by plain Surfaces: treateth in this, of Bodies inclosed under curved Surfaces, as are the Cylinder, the Cone, and the Sphere, and comparing them with each other, he giveth
the

the Rules of their Solidity, and the way of measuring of them. This Book is very useful, seeing we find herein those Principles whereon the most Skilful Geometricians have established so many curious Demonstrations on the Cylinder, the Cone, and of the Sphere.

PROPOSITION I.
THEOREM.

Like Polygons inscribed in a Circle, are in the same Ratio as are the Squares of the Diameters of those Circles.

Plate VIII.

If the Polygons ABCDE, GFHKL, inscribed in Circles, are like, they shall be in the same Ratio as are the Squares of the Diameters AM, FN. Draw the Lines BM, GN, FH, AC.

Demonstration. It is supposed that the Polygons are like, that is to say that the Angles B, and G, are equal; and that there is the same Ratio of AB to BC, as of FG to GH, whence I conclude (by the 6th. of the 6th) that the Triangles ABC, FGH, are equiangular, and that the Angles ACB, FHG, are equal; so then (by the 21st of the 3d) the Angles AMB, FNG, are equal. Now the Angles ABM, FGN, being in a semicircle, are Right (by the 31st of the 3d) and consequently the Triangles ABM, FGN,

FGN, are equiangular, Therefore (by the 5th. of the 6th.) there shall be the same Ratio of AB to FG, as of AM to FN; and (by the 22d. of the 11th.) if one describe Two similar Polygons on AB, FG, which are those proposed; and also Two other like Polygons AM, and FN, which shall be their Squares, there shall be the same Ratio of the Polygon ABCDE to the Polygon FGNKL, as of the Square of AM to the Square of FN.

This Proposition is necessary to Demonstrate the following Proposition.

L E M M A.

IF a Quantity be less than a Circle, there may be inscribed in the Circle, a Regular Polygon, greater than that quantity.

Let the Figure A be less than the Circle B; there may be inscribed in the Circle a Regular Polygon greater than the figure A. Let the figure G, be the difference of the figure A, and the Circle in such sort that the figures A and G taken together be equal to the Circle B. Inscribe in the Circle B, the Square CDEF. (by the 6th. of the 4th.) if that Square were greater than the figure A, we should have what we pretend to. If it be less,

Lemma.
Fig. II.

divide

divide the Quadrants of the Circle CD, DE, EF, FC, into Two equally in the Points H, I, K, L, in such sort as you may have an Octagon. And if that Octagon be less than the figure A; subdivide the Arks, and you shall have a Polygon of Sixteen Sides, then Thirty Two of Sixty Four, &c. I say in fine you shall have a Polygon greater than the figure A; that is to say, a Polygon having less difference from the Circle than the figure A, in such sort that the difference shall be less than the figure G.

Demonstration. The Square inscribed is more than the one half of the Circle, it being the half of the Square described about the Circle, and in describing the Octagon, you take away more than the half of the Remainder, that is to say, the four Segments CHD, DIE, EKF, CLE. For the Triangle CHD, is the half of the Rectangle CO, (by the 34th. of the 1st.) it is then more than the half of the Segment CHD; it is the same of the other Arks. In like manner, in describing a Polygon of Sixteen Sides, you take away more than one half of that which remains in the Circle; and so of the rest. You will then at last leave a lesser quantity than G. For it is evident that having proposed Two unequal Quantities, if you
take

take away more than the half of the greater; and again more than the half of the remainder, and again more than the half of that remainder, and so forward; that which remains shall be less than the second quantity. Let us suppose that the second is contained One Hundred times in the first; it is evident that by Dividing the first into One Hundred parts, in such sort that the first may have a greater Reason to the second, than of Two to One; the second shall be less than the Hundredth part. So then you shall at length meet with a Polygon which shall be less surpassed by the Circle, than the Circle doth the figure A; that is to say, that which remains of the Circle, having taken away the Polygon, shall be less than G. The Polygon shall be greater than the figure A.

PROPOSITION II. THEOREM.

Circles are in the same Ratio as are the Squares of their Diameters.

I Demonstrate that the Circles A and B are in the same Ratio as are the Squares of CD, EF. For if they were not in the same Ratio, the Circle A would have a greater Ratio to the Circle B, than the Square of CD to the Square

Square of EF. Let the figure G have the same Ratio to the Circle B, as hath the Square of CD to the Square of EF: the figure G shall be lesser than the Circle A; (*by the preceding Lemma*) there may be inscribed a Regular Polygon greater than G in the Circle A. Let there also be inscribed in the Circle B, a like regular Polygon.

Demonstration. The Polygon A to the Polygon B, hath the same Ratio, as the Square of CD to the Square of EF; that is to say, the same as hath G to the Circle B: Now the quantity G is lesser than the Polygon inscribed in A: so then (*by the 14th of the 5th*) the Circle should be less than the Polygon which is inscribed, which is evidently false. It must then be said, that the figure G, less than the Circle A, cannot have the same Ratio to the Circle B, as the Square of CD to the Square of EF; and by consequence, that Circle A hath not a greater Ratio to the Circle B, than the Square of CD to the Square of EF. Neither hath it less, because that the Circle B to the Circle A, would have a greater Ratio, and there would be applied the same Demonstration.

Coroll. 1. Circles are in duplicate Ratio of that of their Diameters, because

cause that Squares being like or similar are in duplicate Ratio of that of their sides.

Coroll. 2. Circles are in the same Ratio, as are the similar Polygons inscribed in them.

Coroll. 3. This general Rule must be well taken notice of, when like figures inscribed in other like figures, in such sort that they become nearer and nearer, and degenerate, in fine, into those figures, they are always in the same Ratio. I would say, that if like regular Polygons be described in several Circles, they are in the same Ratio, as are the Squares of the Diameters; and that the greater number of sides they are made to have, they become so much the nearer the Circles; the Circle shall have the same Ratio as the Squares of their Diameters. This way of measuring Bodies by inscription is very necessary.

U S E.

THis Proposition is very universal, and is the way of our reasoning on Circles, after the same manner as on Squares. For example, we say (in the 47th. of the first) that in a Rectangular Triangle, that the Square of the Base is equal to the Square of the other sides taken

taken together. We may say the same of Circles; that is to say, that the Circle described on the Base of a Rectangular Triangle is equal to the Circles which both the sides for Diameters, and after this we may augment or diminish a Circle into what Proportion we list. We prove also in Opticks, that the Light decreaseth in duplicate Ratio of that of the distance of the Luminous Body.

PROPOSITION III.

THEOREM.

EVery Pyramid whose Base is Triangular, may be divided into two equal Prisms, which are greater than half of the Pyramid, and into Two equal Pyramids.

There may be found in the Pyramid ABCD, two equal Prisms EBF, EHK, which shall be greater than half the Pyramid. Divide the six Sides of the Pyramid into equal parts in G, F, E, I, H, K, and draw the Lines EG, GF, FE, HI, IK, EK.

Demonstration. In the Triangle ABD, there is the same Ratio of AG to GB, as of AF to FD; seeing they are equal: thence (by the 2d. of the 6th.) GF, BD, are parallels; and GF shall be the half of BD, that is to say, equal to BH. In like manner GE, BI; FE, HI, shall be parallel and equal; and

(b)

(by the 15th of the 11th) the Planes GFE, BHI, shall be parallel; and by consequence EBFI, shall be a Prism, I say the same of the Figure HEKF, which shall also be a Prism equal to the former; and (by the 40th of the 11th) seeing the Parallelogram Base HIKD, is double to the Triangle BHI, (by the 41st of the first.)

Secondly, The Pyramids AEEG, ECKI, are like an equal.

Demonstration. The Triangles AFG, FDH, are equal (by the 3d of the first) as also EDH, EIK. In like manner the Triangles AGE, EIC, and so of the other Triangles of the Pyramid; they are then equal (by the 10th and 11th def.) they are also similar to the great Pyramid ABDC; for the Triangles ABC, AGE, are like (by the 2d of the 6th) the Lines GE being parallel; which I could demonstrate in all the Triangles of the lesser Pyramids.

In fine, I conclude that the Prisms are more than the one half of the first Pyramid. For if each were equal to one of the lesser Pyramids, the two Prisms would be the half of the great Pyramid. Now they are greater than one of the Pyramids; as the Prism GHE, contains a Pyramid AGFE, which may easily be proved

proved from the *parallelism* of their Sides; whence I conclude that the two Prisms taken together, are greater than the two Pyramids, and consequently greater than half the greater Pyramid.

PROPOSITION IV.

THEOREM.

IF two Triangular Pyramids of equal height, be divided into two Prisms and two Pyramids, and that the last Pyramids be divided after the same manner, all the Prisms of the one Pyramid shall have the same Ratio to all those of the other, as the Base of the one Pyramid hath to the Base of the other.

If one divide the two Pyramids ABCD, DEFG, of equal height, and of Triangular Bases, into two Prisms and into two Pyramids, according to the method of the third Proposition, and if one should subdivide after the same manner the two little Pyramids, and so consecutively, in such sort that there be as many divisions in the one as in the other, there then being the same number of Prisms in both. I say that all the Prisms of the one, to all the Prisms of the other, shall be in the same Ratio as are the Bases.

Demonstration. Seeing the Pyramids are

are of the same height, the Prisms produced by the first division, shall also have the same height, seeing each hath the half of that of their *Pyramids*: Now *Prisms* of equal height, are in the same Ratio as are their Bases (*by the Coroll. of the 32d of the 11th*) the Bases *BTV*, *EPX*, are like unto the Bases *BDC*, *EGF*, and having for sides the half of those of the great Bases, they shall be but the one fourth of the great Bases; so then they shall be in the same Ratio as are the great Bases. Therefore the first Prisms shall have the same Ratio as the greater Bases. I prove after the same manner, that the Prisms produced by the second Division, that is to say of the little *Pyramids*, shall be in the same Ratio, as the greater Bases. Therefore all the Prisms of the one, have the same Ratio to all the Prisms of the other, as the Base to the Base.

U S E.

These two Propositions were necessary to compare *Pyramids* the one with the other, and to measure them.

P

PRO.

PROPOSITION

THEOREM

Triangular Pyramids, of the same height, are in the same Ratio as their Bases.

The Pyramids $ABCD$, $EFGH$, are in the same Ratio as are their Bases. For if they were not, one of the Pyramids, for example $ABCD$, would have a greater Ratio to the Pyramid $EFGH$, than the Base BCD to the Base FGH . So that a quantity L , less than $ABCD$, would have the same Ratio to the Pyramid $EFGH$, as the Base BCD to the Base FGH . Divide the Pyramid $ABCD$ after the manner of the Third Proposition; divide also the Pyramid, which will result of the first division into two Prisms, and into two Pyramids; and those again into two other Prisms, continue dividing after this manner as long as need requires: Seeing the Prisms of the first division are more than the half of the Pyramid $ABCD$ (by the 3d) and that the Prisms of the second Division is greater than half the Remainder; that is to say than the little Pyramid, and that those of the third Division is more than the half Remainder; it is evident, that by making many subdivisions,

visions, there will at length remain a quantity less than the excess of the Pyramid ABCD above the quantity L; that is to say, that all the Prisms being put together, shall be greater than the quantity L. Make as many Divisions in the Pyramid EFGH, until you have as many Prisms as there are in ABCD.

Demonstration. The Prisms of ABCD hath the same Ratio to the Prisms of EFGH, as hath the Base BCD to the Base FGH. Now the Ratio of the Base BCD to the Base FGH, is the same with that of the quantity L to the Pyramid EFGH. There is therefore the same Ratio of the Prisms of ABCD to the Prisms of EFGH, as of the quantity L to the Pyramid EFGH. Again, the Prisms of ABCD are greater than the quantity L; thence (*by the 14th of the 5th*) the Prisms comprehended in the Pyramid EFGH, would be greater than the same Pyramid EFGH; which is evidently false, the part not being capable of being greater than the whole. We must then acknowledge that a Magnitude less than one of the Pyramids, cannot have the same Ratio to the other, as hath the Base to the Base; and consequently, neither of the Pyramids hath a greater Ratio to the other, than the Base to the Base. P 2. PRO.

PROPOSITION VI. THEOREM.

ALL sorts of Pyramids, of the same height, have the same Ratio as their Bases.

The Pyramids ABC, DEFG, of the same height, are in the same Ratio as are their Bases BC, EFG. Divide the Bases into Triangles.

Demonstration. The Triangular Pyramids AB, DE, of the same height, are in the same Ratio as are their Bases (*by the 5th.*) In like manner, the Triangular Pyramids AC, DF, are in the same Ratio as their Bases. There shall be then the same Ratio of the Pyramid ABC to the Pyramid DEF, as of the Base BC to the Base EF (*by the 12th of the 5th.*) Again, seeing there is the same Ratio of the Pyramid DEF to the Pyramid ABC, as of the Base EF to the Base BC; and that there is also the same Ratio of the Pyramid DG to the Pyramid ABC, as of the Base G to the Base BC; there will be also the same Ratio of the Pyramid DEFG to the Pyramid ABC, as of the Base EFG to Base BC.

PRO-

PROPOSITION VII.
THEOREM.

EVery Pyramid is the third part of a Prism of the same Base, and of the same height.

Let there be proposed in the first place a Triangular Prism AB; I say that a Pyramid which hath for its Base one of the Triangles ACE, BDF, and which shall be of the same height, as the Pyramid ACEF, shall be the third part of the Prism. Draw the three Diagonals AF, DC, FC, of the same parallelogram.

Demonstration. The Prism is divided into three equal Pyramids ACEF, ACFD, CFBD; thence each of them shall be the third part of the Prism: The two first, having for Base the Triangles AFE, AFD, which are equal (*by the 34th of the first*) and for height the perpendicular drawn from their top C to the plane of their Base AF, they shall be equal (*by the preceding.*) The Pyramids ACFD, CFBD, which have for Bases equal Triangles ADC, DCB, and the same Vertex F, shall also be equal (*by the preceding.*) Thence one of these Pyramids; for example AFEC, which hath the same Base

P 3
ACE,

ACE, as the *Prism*; and the same height, which would be the perpendicular drawn from the Point F, to the Plane of the Base ACE, is the third part of the same. If the Prism were a Polygon, it must be then divided into several Triangular Prisms; and the Pyramid which would have the same Base, and the same height, would be also divided into so many Triangular Pyramids, each of which would be the third part of its Prism. Therefore (*by the 32d of the 5th*) the Polygonal Pyramid would be the third part of the Polygonal Prism.

PROPOSITION VIII.

THEOREM.

Like Pyramids are in triple Ratio of their homologous Sides.

If the Pyramids are Triangular, they may be converted into Prisms, which shall be also like, seeing they would have some Planes, which would be the same as those of the Pyramids. Now like Prisms are in triple Ratio of their homologous sides (*by the 39th of the 11th*) thence the Pyramids which are the third of them (*by the preceding*) shall be in triple Ratio of their homologous sides.

If

If the Pyramids are Polygons, they may be reduced, into Triangular Pyramids.

PROPOSITION IX.

THEOREM.

Equal Pyramids have their Heights and Bases reciprocal, and those Pyramids whose heights and Bases are reciprocal, are equal.

Let there be proposed two Triangular Pyramids, they shall have their Bases and heights reciprocal. Let there be made Prisms of equal heights and Bases; seeing those Prisms are triple each to its Pyramid (by the 7th) they shall be also equal. Now equal Prisms, have their Bases and heights reciprocal (by the 4th Coroll. of the 39th) thence the Bases and heights of the Pyramids, which are the same with those of the Prisms shall be reciprocal to them.

Secondly, if the Bases and heights of Pyramids be reciprocal, the Prisms shall be equal, as also the Pyramids, which are the third parts of them.

If the Pyramids were Polygons, they ought to be reduced into Triangular Pyramids.

Coroll. There might be made some other Propositions; for example, that

Pyramids of equal height, are in the same Ratio as their Bases, and those which have the same Base, are in the same Ratio as their heights.

U S E.

From these Propositions is gathered a way of measuring of Pyramids, which is, to multiply the Base by the one third of its height. Then there might be made some other Propositions, as that if a Prism be equal to a Pyramid, the Bases and height of the Prism, with the third part of the height of the Pyramid shall be reciprocal; that is to say, if there be the same Ratio of the Pyramid to the Base of the Prism, as of the height of the Prism to the third part of the height of the Pyramid; the Pyramid and Prism shall then be equal.

L E M M A.

IF there be proposed a quantity less than a Cylinder, there may be inscribed in that Cylinder a Polygon Prism greater than that quantity.

If the quantity *A* be less than the Cylinder which hath the Circle *B* for its Base; there may be inscribed in the Cylinder a Polygon Prism greater than the quantity *A*. The Square *CDEF* is inscribed,
GHIK

GH $\mathcal{K}$ is circumscribed, CLDMENFO is an Octagon. Draw the Tangent PL $\mathcal{Q}$, and continue the sides ED, FC, to P and $\mathcal{Q}$; and imagine as many Prisms of the same height as the Cylinder, which have for their Bases those Polygons. That which hath for its Base the Circumscribed square incompasseth the Cylinder, and that on the inscribed Square is also inscribed in the Cylinder.

Demonstration. The Prisms which have the same height, are in the same Ratio as are their Bases (by the Coroll. of the 39th of the 11th) and the inscribed Square being the half of the circumscribed Square; its Prism shall be the half of the Cylinder. Making a Prism which hath the Octagon for Base, there will be taken away more than the half of that which remained for the Cylinder, having taken away the Prism of the inscribed Square; because the Triangle CLD is half the Rectangle C $\mathcal{Q}$; and because that Prisms of equal or the same height, are in Ratio as are their Bases, the Prism which hath for its Base the Triangle CLD, shall be the half of the Prism of the Rectangle DCP $\mathcal{Q}$; it shall then be more than the half of the part of the Cylinder, which hath for Base the Segment DLC. It is the same of the other Segments. It

demonstrate after the same manner, that in making a Polygon Prism of sixteen sides, I take away more than half of what remained of the Cylinder, by taking away the Octagonal Prism; there will remain a part of the Cylinder, less than the excess of the Cylinder above the quantity A. We shall then have a Prism inscribed in the Cylinder, which will be less exceeded by the Cylinder, than the quantity A. One might argue after the same method about inscribed Pyramids in a Cone.

PROPOSITION X.

THEOREM.

A Cone is the third part of a Cylinder of the same Base, and the same height.

If a Cone, and a Cylinder have the Circle A for Base, and the same height; the Cylinder shall be triple to the Cone. For if the Cylinder had a greater Ratio to the Cone, than the triple; a quantity B less than the Cylinder, would have the same Ratio to the Cone, as three to one; and (by the preceding Lemma) there might be inscribed in the Cylinder, a Polygonal Prism greater than the quantity B. Let us suppose that that quantity is that which hath for

for Base the Polygon GDEFGH. Make also on the same Base, a Pyramid inscribed in the Cone.

Demonstration. The Cylinder, the Cone, the Prism, and the Pyramid are all of the same height; thence the Prism is triple to the Pyramid (*by the 7th.*) Now the quantity B is also the triple of the Cone: there is then therefore the same Ratio of the Prism to the Pyramid, as of the quantity B to the Cone; and (*by the 14th of the 5th*) seeing the Prism is greater than the quantity B, the Pyramid should be greater than the Cone in which it is inscribed, which cannot be.

But if it were said that the Cone hath a greater Ratio to the Cylinder, than one to three; there might be taken the same method to demonstrate the contrary.

PROPOSITION XI.

THEOREM.

Cylinders and Cones of the same height, are in the same Ratio as are their Bases.

There is proposed two Cylinders or two Cones of the same height, which have the Circles A and B for their Bases:

ses: I say they are in the same Ratio as are their Bases. For if they are not in the same Ratio; one of them, for example A, shall have a greater Ratio to the Cylinder B, than the Base A to the Base B; so that a quantity L, less than the Cylinder A, would have the same Ratio to the Cylinder B, as hath the Base A to the Base B. There may then be inscribed a Polygon Prism in the Cylinder A, greater than the quantity L. Let it be that which hath for Base the Polygon CDEF; and let there be inscribed a like Polygon GHIK, in the Base B, which serveth for Base to the Cylinder of the same height.

Demonstration. The Prisms A and B are in the same Ratio as are their Polygon Bases (*by the Coroll. 4. of the 39th of the 11th*) and the Polygons are in the same Ratio as are the Circles (*by the Coroll. of the 2d*) so then the Prism A shall be in the same Ratio to the Prism B, as the Circle A to the Circle B. Now as the Circle A is to the Circle B, so is the quantity L to the Cylinder B; therefore as the Prism A is to the Prism B, so is the quantity L to the Cylinder B. The Prism A is greater than the quantity L; by consequence (*according to the 14th of the 5th*) the Prism B inscribed in

in the Cylinder B, would be greater than it, which cannot be. Therefore neither Cylinder hath greater Ratio to the other, than that of its Base to the Base of the other.

Coroll. Cylinders are triple to Cones of the same height, therefore Cones of the same height, are in the same Ratio as are their Bases.

PROPOSITION XII.

THEOREM.

Like Cylinders and Cones are in triple Ratio to that of the Diameters of their Bases.

Let there be proposed two like Cylinders, or two Cones, which have for their Bases the Circles A and B. I say that the Ratio of the Cylinder A to the Cylinder B, is in triple Ratio of the Diameter DC to the Diameter EF. For if it hath not a triple Ratio, let the quantity G, less than the Cylinder A, have to the Cylinder B a triple Ratio of that of the Diameter DC to the Diameter EF; and let there be inscribed in the Cylinder A a *Prism*, which let be greater than G; and another like thereto in the Cylinder B: they shall have the same Altitude or height as had the Cylinders, for like Cylinders have their

their heights and the Diameters of their Bases proportional, as well as the Prisms (by the 11th Def. of the 11th.)

Demonstration. The Diameter DC hath the same Ratio to the Diameter EF, as hath the side DI to the side DL, or DC, to EF (as I demonstrated in the first.) Now the Prisms are in triple Ratio of their homologous Sides (by the Coroll. 4. of the 39 of 11) therefore the Prism A to the Prism B, is in triple Ratio of DC to EF. We have supposed that the quantity G, in respect of the Cylinder B, was in triple Ratio of the Diameter DC to the Diameter EF. So then there will be the same Ratio of the Prism A to the Prism B, as of the quantity G to the Cylinder B; and (by the 14th of the 5th) seeing that the Prism A, is greater than the quantity G; the Prism B, that is to say, described in the Cylinder B, would be greater than the same Cylinder, which is impossible. Therefore like Cylinders are in triple Ratio of the Diameters of their Bases.

Cones are the third parts of Cylinders (by the 10th;) therefore like Cones are in triple Ratio of that of the Diameters of their Bases.

PRO.

PROPOSITION XIII.

THEOREM.

IF a Cylinder be cut by a Plane parallel to its Base, the parts of the Axis shall be in the same Ratio, as are the parts of the Cylinder.

Let the Cylinder AB , be cut by the Plane DC , parallel to its Base. I say that there shall be the same Ratio of the Cylinder AF to the Cylinder FB , as of the Line AF to the Line FB . Draw the Line BG perpendicular to the Plane of its Base A ; draw also in the Planes of the Circles DC , and A , the Lines FB , AG .

Demonstration. The Plane of the Triangle BAG , cutteth the parallel Planes A and DC ; thence the Sections EF , and AG , are parallel (*by the 16th. of the 11th.*) So then there is the same Ratio of AF to FB , as of the height GE to EB . Let there be taken an Aliquot part of EB , and having divided GE , EB , into parts equal to that Aliquot part, let there be drawn Planes parallel to the Base AG : You shall have so many Cylinders of the same height, which having their Bases and heights equal, they shall be equal (*by the 11th.*) Again, the Lines AF and FB , shall be
divi-

divided after the same manner as EG, EB, (*by the 16th. of the first.* So then the Line AF contains as many times the Aliquot part of the Line FB, as the Cylinder AF contains a like Aliquot part of FB. There is thence the same Ratio of the parts of the Cylinder, as of the parts of the Axis.

Coroll. The parts of the perpendicular are in the same Ratio as are the parts of the Cylinder.

PROPOSITION XIV. THEOREM.

C*ylinders and Cones having the same Base, are in the same Ratio as are their height.*

Two Cylinders AB, CD, of equal Bases, being proposed; cut out of the greater a Cylinder of the same height, with the lesser, by drawing a Plane EF parallel to its Base. It is evident that the Cylinders EF, AB, are equal (*by the 11th*) and that CF to CD, hath the same Ratio as GI to GH, or (*by the Coroll. of the preceding*) as the height of the Cylinder CF to the height of CD, there is thence the same Ratio of AB to CD, as of the height of EF or AB, to the height of CD.

As to Cones, seeing they are the one thirds

thirds of Cylinders, if they have equal Bases, they shall be also in the same Ratio as are their height.

PROPOSITION XV.

THEOREM.

E*qual Cylinders and Cones, have their Bases and height reciprocal, and those which have their Bases and heights reciprocal, are equal.*

If the Cylinders AB, CD, are equal; there shall be the same Ratio of the Base B to the Base D, as of the height CD to the height AB. Let the height DE be equal to the height AB.

Demonstration. There is the same Ratio of the Cylinder AB to the Cylinder DE, of the same height, as of the Base B to the Base D (*by the 14th.*) Now as the Cylinder AB is to the Cylinder DE; so is the Cylinder CD equal to AB, to the Cylinder DE; that is to say, so is the height CD to the height AB or DE. Therefore as the Base B is to the Base D, so is the height CD to the height AB.

Secondly, If there be the same Ratio of the Base B to the Base D, as of the height CD to the height AB; the Cylinders AB, CD, shall be equal. For the Cylinder AB is to the Cylinder DE,

DE, as is the Base B, to the Base D, and the Cylinder CD shall have the same Ratio to the Cylinder DE, as CD to DE; there shall therefore be the same Ratio of AB to DE, as of CD to DE: and (by the 8th. of the 5th.) the Cylinders AB, and CD, shall be equal. $\square$

The Propositions 16 and 17 are very difficult, and are only of use in the 18th. which I will Demonstrate after a more easie method by the Two following Lemmas.

LEMMA I.

IF there be proposed a quantity less than a Sphere; there may be inscribed in the same Sphere a number of Cylinders, which taken together shall be of the same height, and greater than that quantity.

Lemma.
Fig. I.

Let ABC be half of a great Circle of the Sphere, and let the quantity D be less than the same hemi-sphere; I say that there may be inscribed therein several Cylinders of the same height, which taken together, shall be greater than the quantity D. For if the hemi-sphere surpasseth in bigness the quantity D, it will surpass it by a certain quantity; let that quantity be the Cylinder MP. In such sort

sort that the quantity D , and MP taken together, be equal to the hemi-sphere. Make or conceive it so made, that there be the same Ratio of a great Circle of the Sphere to the Base MO , as of the height MN to the height R . Divide the Line EB into as many equal parts as you please, each of them less than R ; and drawing parallels to the Line AG , describe inscribed and circumscribed parallelograms. The Number of the circumscribed shall surpass by a unit those of the inscribed. Now all the circumscribed rectangles surpass the inscribed, by the little Rectangles through which the Circumference of the Circle passeth; and all those Rectangles taken together are equal to the Rectangle AL . Imagine that the Semi-Circle is made to roll about the Diameter EB ; the Semi-Circle shall describe a hemi-sphere, and the inscribed Rectangles will describe inscribed Cylinders in the semi-sphere, and the Circumscribed will describe other Cylinders.

Demonstration. The Circumscribed Cylinders surpass more the inscribed, than doth the hemi-sphere surpass the same inscribed Cylinders; seeing that they are comprehended within the Circumscribed Cylinders. Now the Circumscribed surpass the inscribed by so much as is the Cylind-

der AL ; therefore the hemi-sphere shall surpass by less the inscribed Cylinders, than doth the Cylinder made by the Rect-angle AL . The Cylinder AL , is less than the Cylinder MP ; for there is the same Ratio of a great Circle of the Sphere which serveth for Base to the Cylinder AL , as of MP to R ; so then by the foregoing, a Cylinder which hath for Base a great Circle of the Sphere, and the Altitude R , would be equal to the Cylinder MP . Consequently the hemi-sphere which surpasseth the quantity D , by the Cylinder MP ; and the inscribed Cylinders be a quantity less than AL , surpasseth the inscribed Cylinders by less than the quantity D . Therefore the quantity D , is less than the inscribed Cylinders.

What I have said of a hemi-sphere, may be said of a whole Sphere.

LEMMA II.

Like Cylinders inscribed in Two Spheres, are in Triple Ratio of the Diameters of the Spheres.

Lemma.
Fig. II.

If the two like Cylinders CD , EF , be inscribed in the Spheres A , B ; they shall be in Triple Ratio of the Diameters LM , NO . Draw the Lines GD , IE .

Demonstration. The like Right Cylinders CD , EF , are like; so then there

is the same Ratio of HD to DR, as of QF to FS; as also the same Ratio of KD to KG, as of PF to PL. And consequently the Triangles GDK, IFP, are like (by the 6th. of the 6th.) so there shall be the same Ratio of KD to PF, as of GD to IF, or of LM to ON. Now the like Cylinders CD, EF, are in Triple Ratio of KD and PF, the Diameters of their Bases (by the 12th.) therefore the like Cylinders CD, EF, inscribed in the Spheres A and B, are in triple Ratio of the Diameters of the Spheres.

PROPOSITION XVIII.

THEOREM.

Spheres are in triple Ratio of their Diameters.

The Spheres A and B are in Triple Ratio of their Diameters CD, EF. For if they be not in Triple Ratio; one of the Spheres, as A, shall be in a greater Ratio than Triple, of that of CD to EF; therefore a quantity G less than the Sphere A, shall be in Triple Ratio of that of CD to EF; and so one might (according to the first Lemma) inscribe in the Sphere A, Cylinders of the same height, greater than the quantity G.

Let there be inscribed in the Sphere B, as many like Cylinders as those of the Cylinder A.

De-

Demonstration. The Cylinders of the Sphere A, to those of the Sphere B, shall be in Triple Ratio of that of CD to EF (by the preceeding.) Now the quantity G in respect of the Sphere B, is in Triple Ratio of CD to EF; there is then the same Ratio of the Cylinders of the Sphere A, to the like Cylinders of the Sphere B. So then the Cylinders of A being greater than the quantity G; the Cylinder B, that is to say, inscribed in the Sphere B, would be greater than the Sphere B, which is impossible. Therefore the Spheres A and B are in Triple Ratio of that of their Diameters.

Coroll. Spheres are in the same Ratio as are the Cubes of their Diameters, seeing that the Cubes being like Solids, are in Triple Ratio of that of their Sides. EF, CD

Ratio of their Diameters CD, EF. CD

Ratio of their Diameters CD, EF. CD

Ratio of their Diameters CD, EF. CD

Ratio of their Diameters CD, EF. CD

Ratio of their Diameters CD, EF. CD

Ratio of their Diameters CD, EF. CD

Ratio of their Diameters CD, EF. CD

Plate

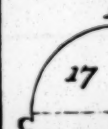
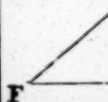
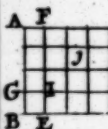
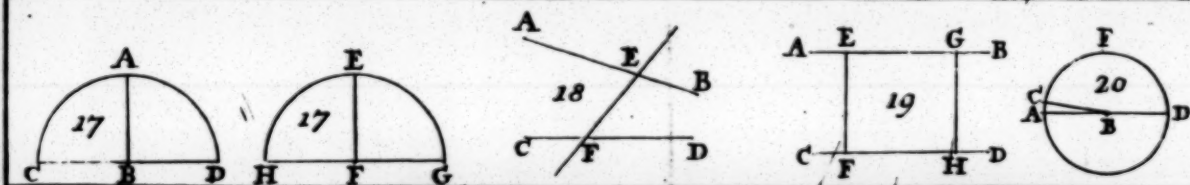
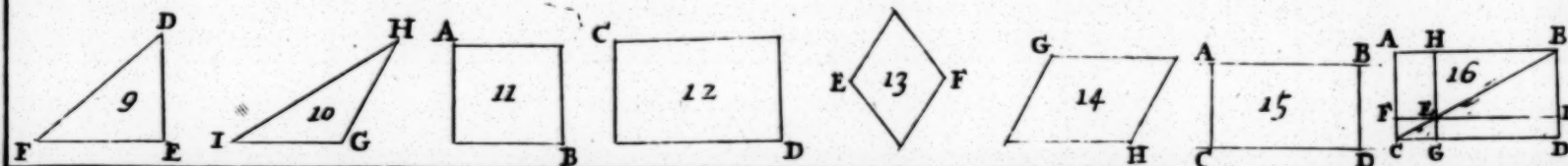
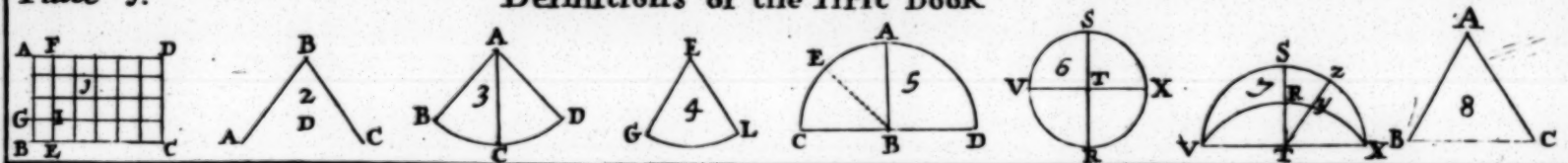
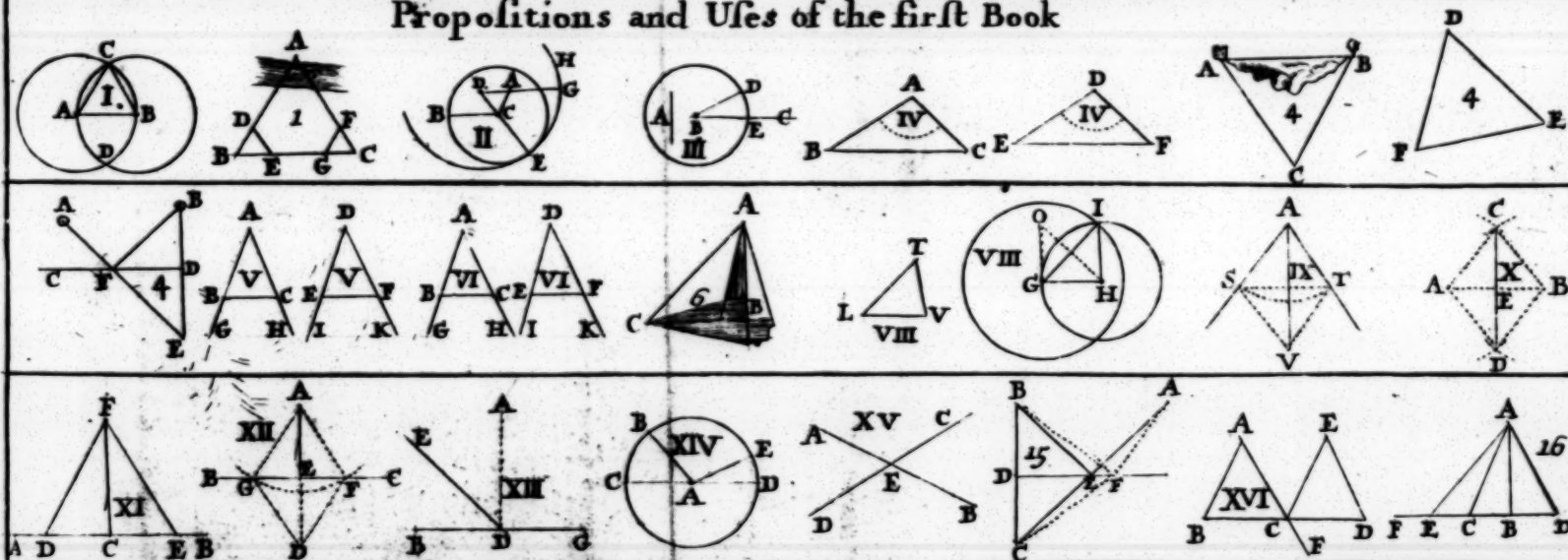


Plate 3.

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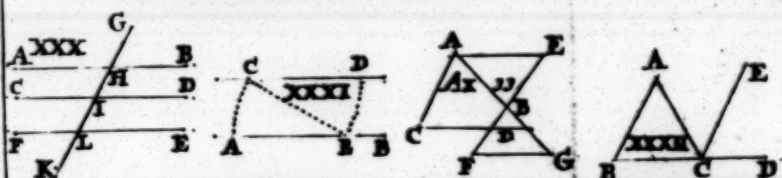
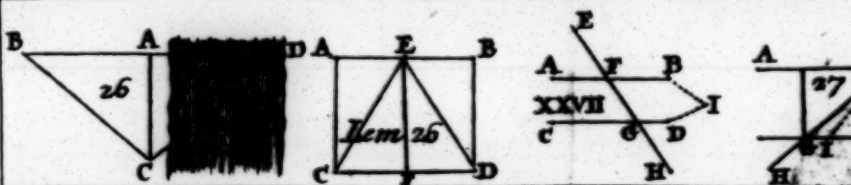
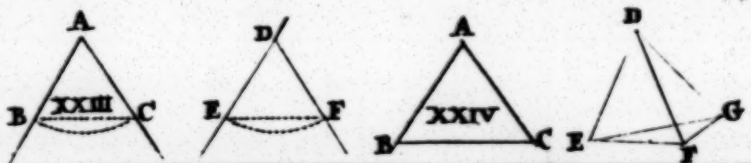
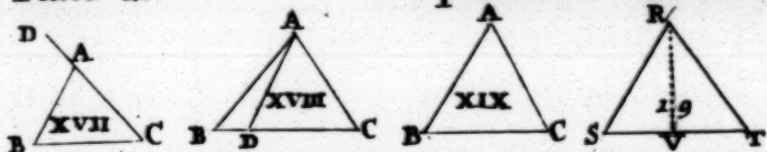
Propositions and Uses of the first Book



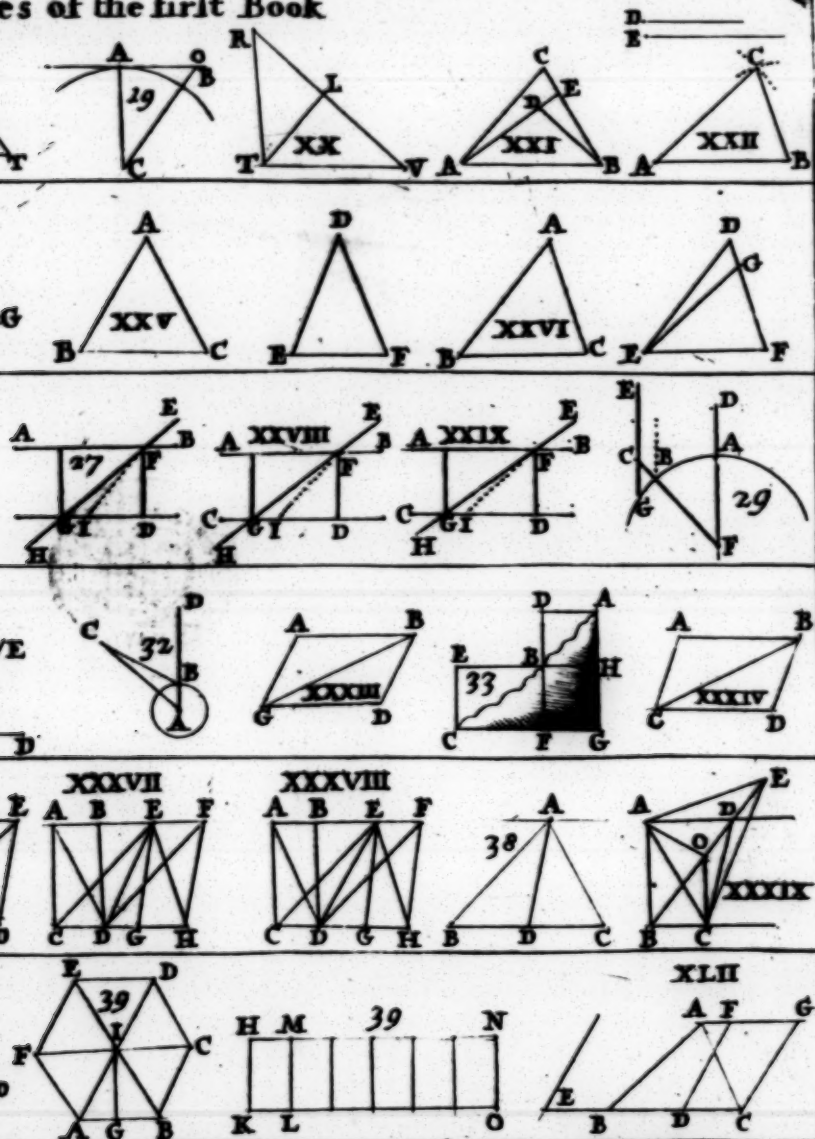
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Plate 2.

Propositions and Uses of t

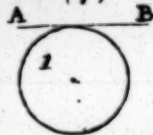


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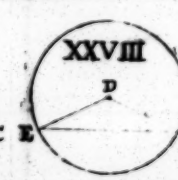
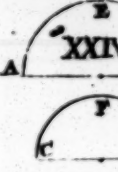
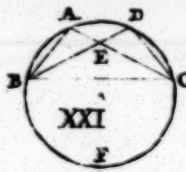
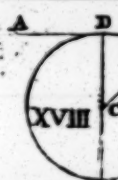
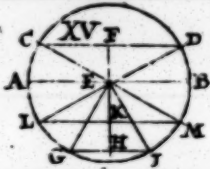
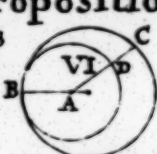


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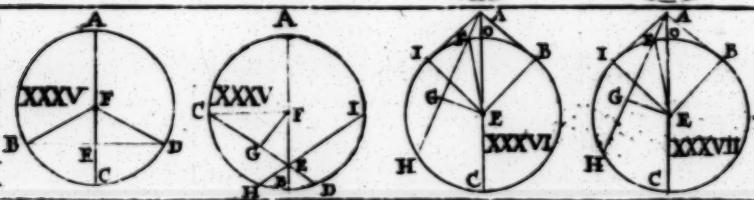
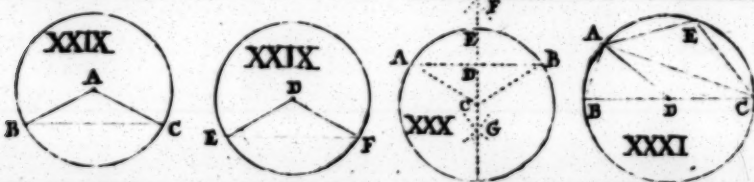
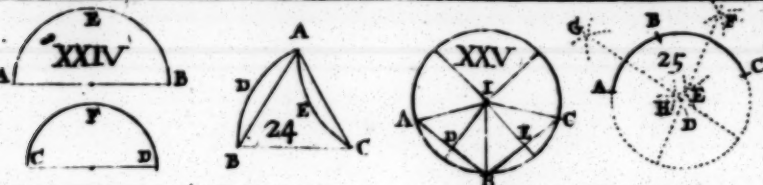
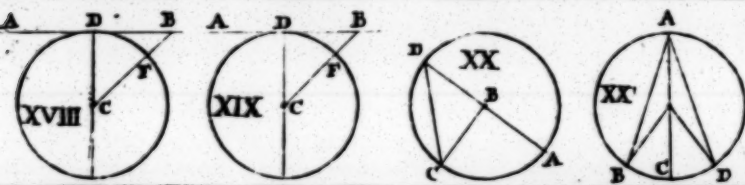
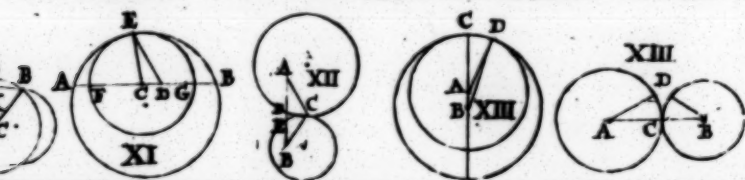
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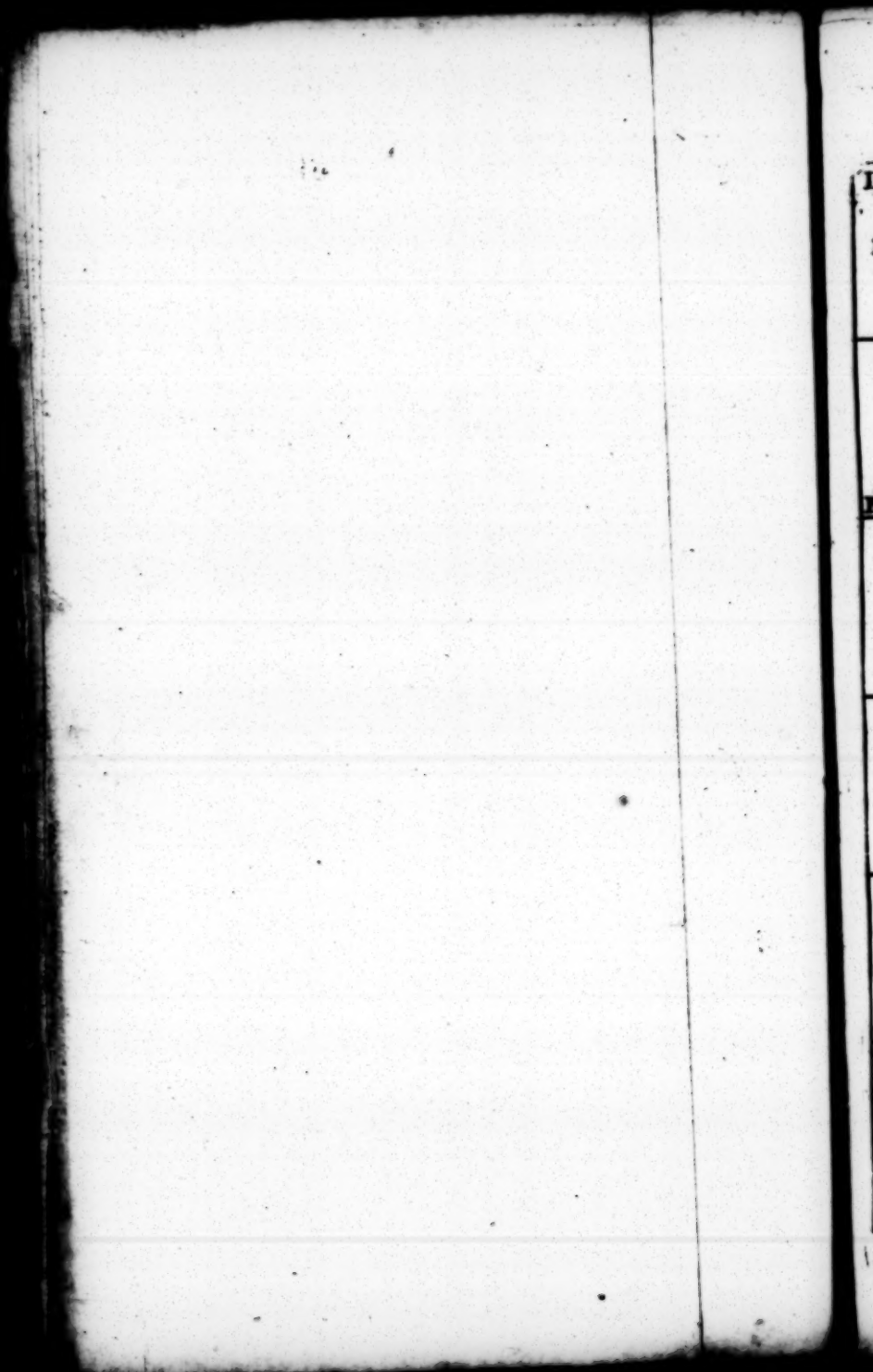


Propositions and Uses



Propositions





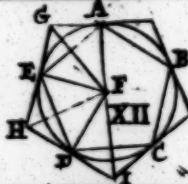
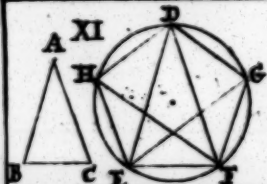
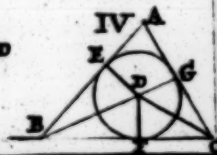
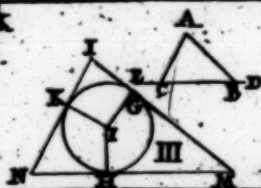
Definitions



Propositions of



the fourth Book



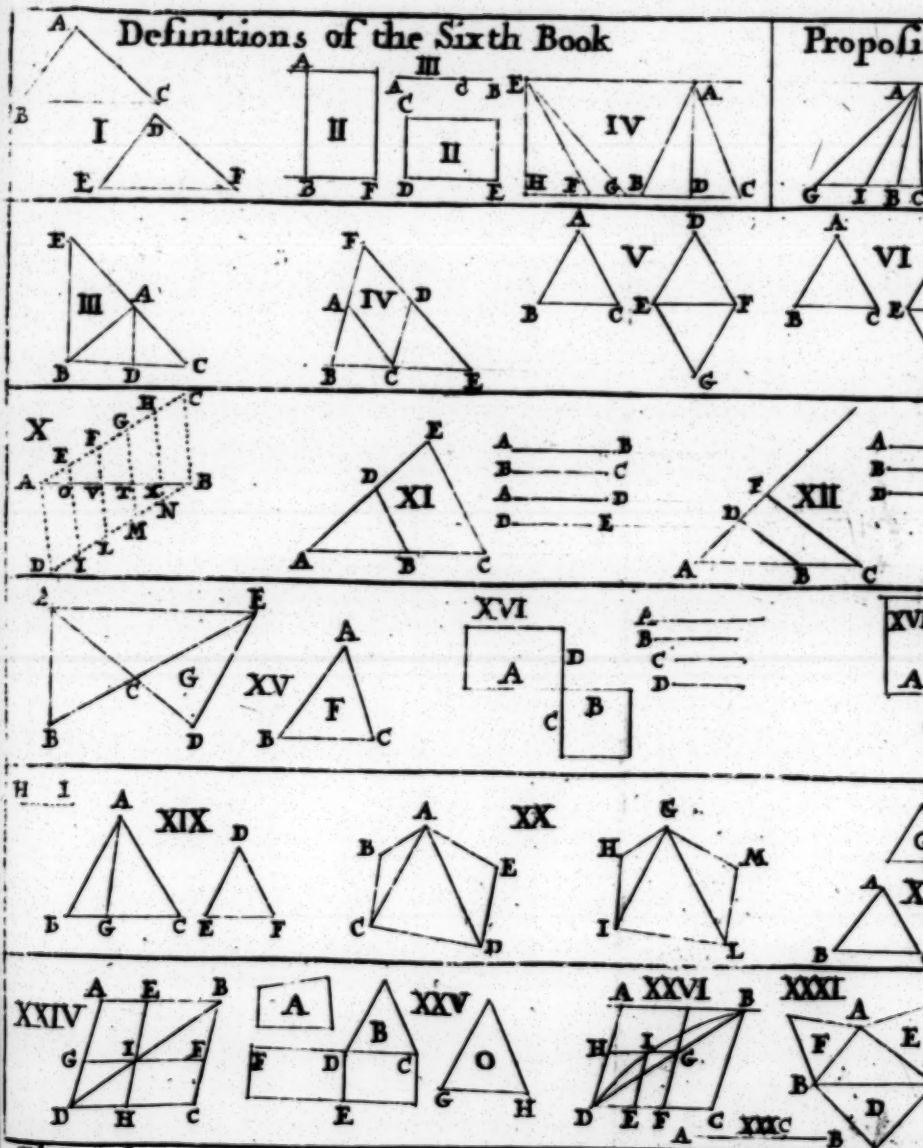
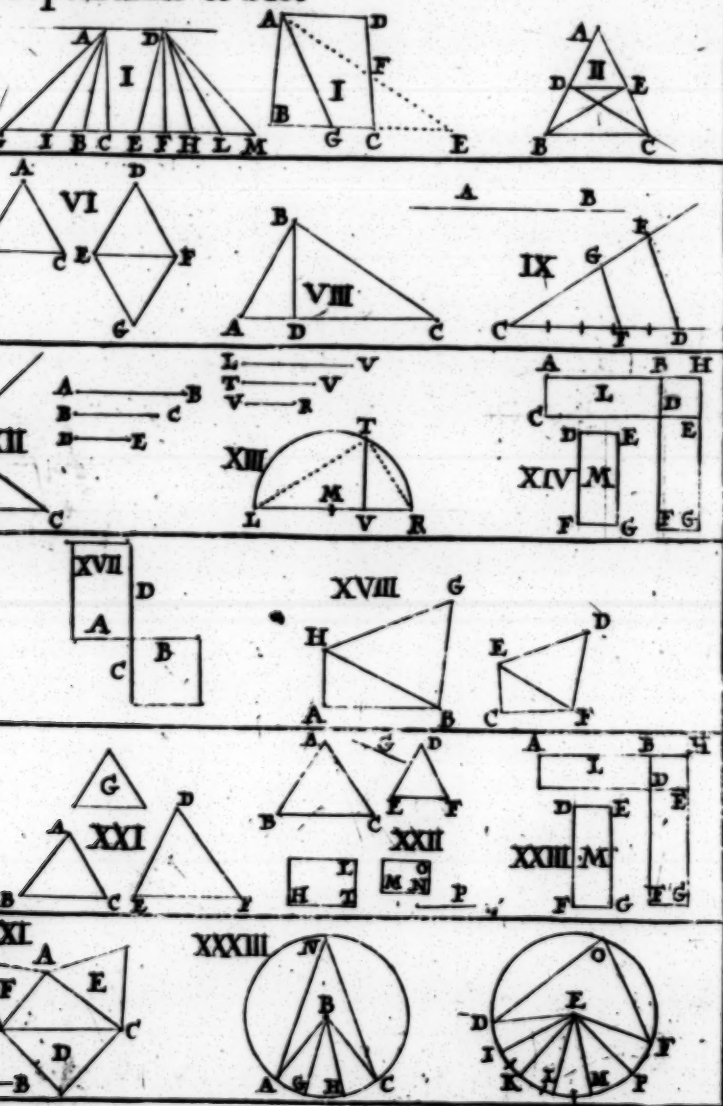


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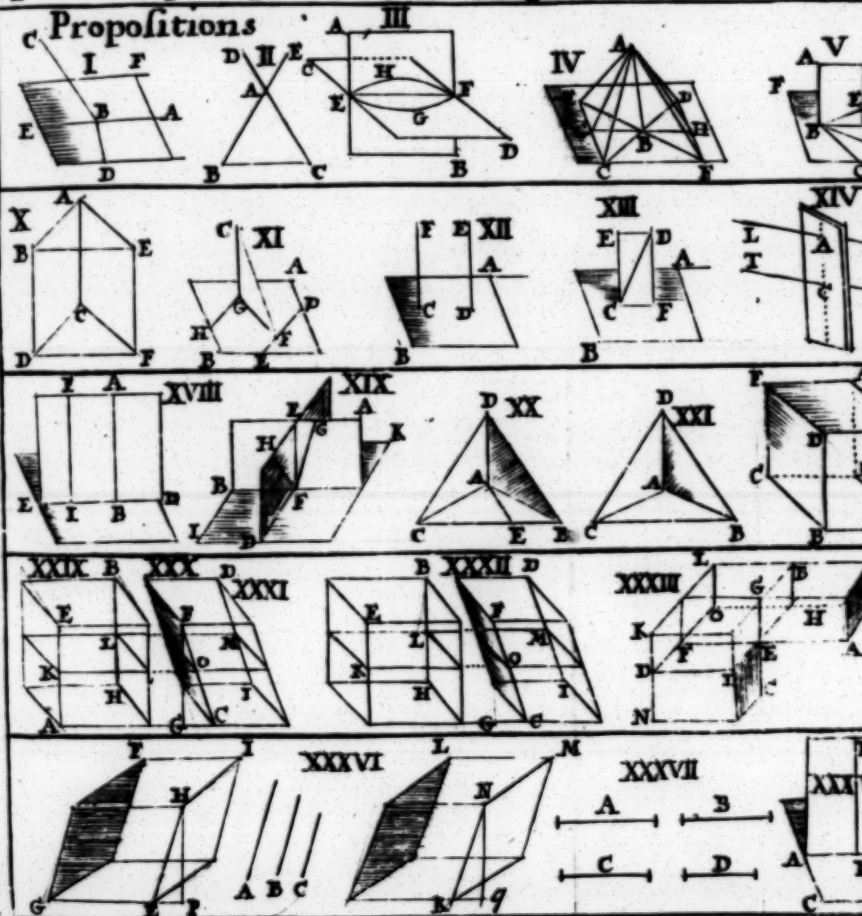
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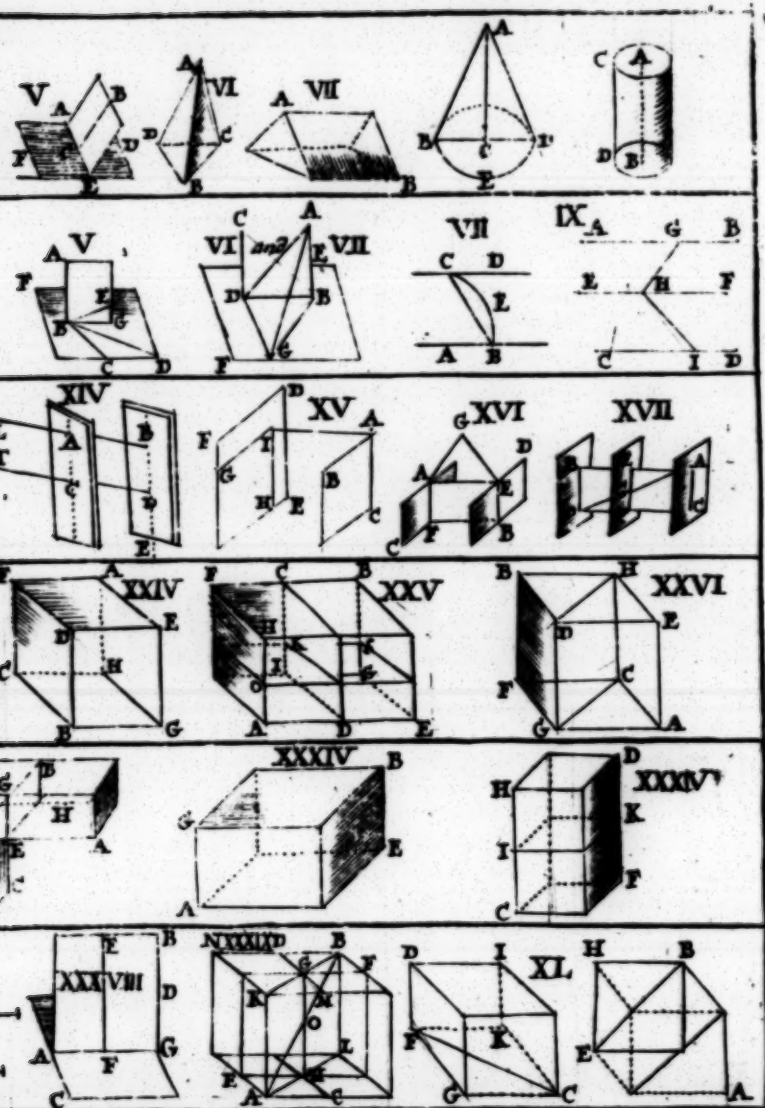


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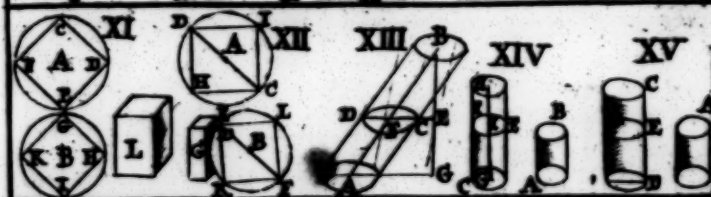
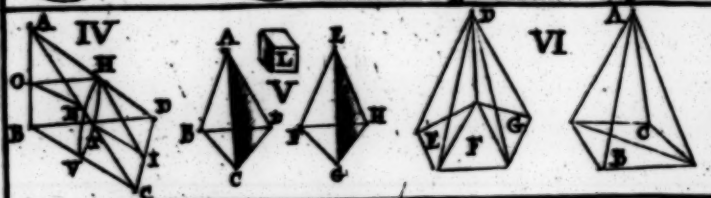
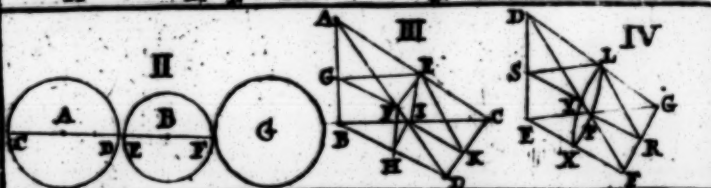


Propositions



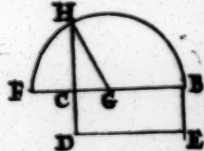
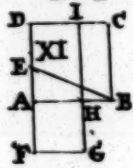
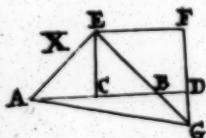
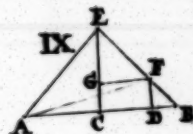
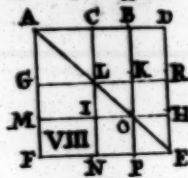
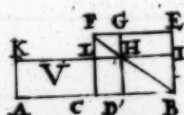
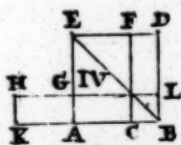
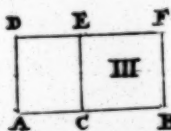
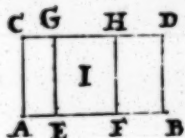
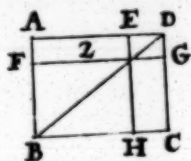


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